

# Solar Radiation Pressure Effects on the Orbital Elements of Artificial Earth Satellite

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## Abstract:

The solar radiation pressure is one of the non-conservative force perturbation effects on earth satellite. A method presented to calculate the effect of radiation pressure on orbital elements for different altitude of satellites, the necessary calculation down by using modern Mat lab computer language. Typical results presented for radiation effect at deferent values of area mass ratio( $A/m$ ), the computed results indicated that the orbital elements much affected by solar radiation at high value of area mass ratio, also the effect on high orbit satellite is much more than that on low orbit satellite. The study also include the solar radiation pressure at two different value of ascending node ( $\Omega = 0^\circ$  and  $\Omega = 180^\circ$ ).

**Key word:** solar radiation pressure, area mass ratio, orbital elements, Earth shadow, perturbations

## 1. Introduction:

Perturbations are deviations from a normal idealized or unperturbed motion [1]. The usual perturbation for the orbit problem are the asphericity of the central body, atmospheric drag and left, the presence of other attracting bodies, effect of solar radiation pressure, thrust, magnetic fields, solid earth tides, ocean tides, Earth re-radiation and relativistic effect[1]. Several authors have considered the effect of solar radiation pressure on the motion of an artificial satellite; Brooks C.J. and Ryland F.C.E.[2] observed the perturbations in orbital elemental that caused by diffuse radiation pressure. Moore P.[3] study the perturbation on spheroid satellite using the variation of the approach developed by Aksnes K.[4]. Saha L.M.[5] investigates the rotational motion of a satellite influenced solar radiation pressure and tidal force. With the advent of satellites of large area to mass ratio, the magnitude of the radiation pressure effect is substantial and must be taken into account in the analysis of the tracking data [6, 7, 8]. The central object in this paper is to present the effects of radiation pressure on the orbital elements at different area to mass ratio and different reflectivity.

In fact many interplanetary missions would miss their target entirely if the perturbing effect of other attracting bodies weren't taken into account. Ignoring the effect of the central body oblateness on any satellite keeps us from accurately predicting its position over a long time. There are three main approaches to examine the effect of perturbations, special perturbation techniques (using numerical methods), general perturbation techniques (using analytical methods), and semi analytical techniques.

The forces causing the perturbative effects on the satellite are either conservative or non-conservative solar radiation pressure is examples of non-conservative force. It's common to analyze perturbations using a disturbing function and a disturbing force. The non conservative force of solar radiation pressure is usually modeled as a disturbing force. Solar radiation pressure is a non-conservative perturbation, but it becomes important at higher altitudes. One of the more difficult aspects of

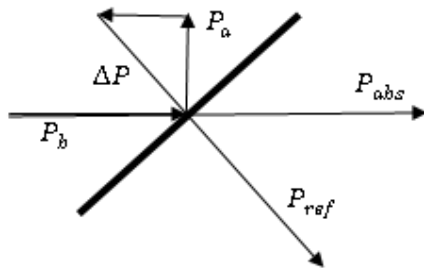
analyzing solar radiation is accurately modeling and predicting the solar cycles and variations. During periods of intense solar storms, this effect may be much larger than all the other perturbations (depending on the altitude); at times of low activity, the effect may be negligible [1]. The same difficulties arise for cross-sectional area as for drag; however, solar-radiation pressure also requires us to determine the shadowing effect on the spacecraft itself which discussed in section 4. To examine the overall effect of solar radiation, we must first develop expressions for the specific force (acceleration) and how it is measured. Because the incoming radiation from the Sun causes a force on the satellite, the apparent size of the satellite that faces the Sun is crucial in accurately determining the amount of acceleration. The pressure is simply the force divided by the incident area exposed to the Sun. This means that the pressure distribution is very critical, and this depends on the satellite's shape and composition determining the Sun's precise location; the correct orbital attitude; the exact value of the solar-radiation pressure; the effective, time-varying, cross-sectional area exposed to the incoming radiation (the solar radiation pressure or force is towards the satellite from the sun it is inversely proportional to the mass of the satellite i.e. if the satellite is light and large then it is more affected.); and the correct and usually time-varying coefficients to model the satellite's reflectivity[1]. The satellite is constructed of materials that carry different refectories therefore modulation of such perturbation is very complicated[9].

## 2. Solar Radiation Pressure

According to quantum mechanics theory, each photon of frequency  $f$  and wavelength ( $\lambda = \frac{c}{f}$ ) where  $c$  is the speed of light in vacuum carries the energy ( $E = hf$ ) and the linear momentum

( $P_l = \frac{hf}{c} \hat{e}$ ) where  $h = 6.62 * 10^{-34} J.sec$  Planck constant.  $\hat{e}$  is the unit vector of the propagation of the photon.

The momentum transferred per time unit onto unit surface in a radiation field is called radiation pressure [10].



**Figure (2.1) Absorption and reflection of radiation by the surface [10].**

According to the law of conservative of linear momentum ( $-\Delta P_i$ ) is the momentum gained by the surface elements. If the photon is absorbed by the surface, ( $P_a = 0$ ) and the surface gain the momentum ( $P_{abs} = P_b$ ). The surface gain linear momentum to its surface with absolute value  $2|P_b|\cos\alpha = |P_{ref}|$ .

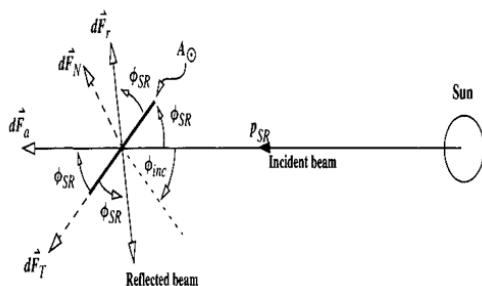
The primary radiation source to be considered in satellite geodesy is the sun. The radiation pressure due to the direct solar radiation is also referred to as direct solar radiation [10]. Other sources of radiation are the Earth, which reflects or reemits the radiation received by the sun, or much lesser extent, the Moon, reflecting the solar radiation. If 100% of the radiation is absorbed, the linear momentum  $P_l = \frac{S}{c} \hat{e}$  gained in one second by the surface elements of  $1 \text{ m}^2$  normal to the direction sun, surface element at 1AU.

Where  $S = 1368 \text{ Watt/m}^2$  is the solar constant giving the energy flowing through the surface per time unit at the distance of one astronomical unit. The force exerted by the solar radiation pressure on the satellite given by[1]:

$$F_{SR} = P_{SR} C_R A \quad \dots \dots \dots (1)$$

Where the reflectivity  $C_R$  is a value between 0.0 and 2.0 indicates how the satellite reflects incoming radiation.  $P_{SR}$  is the solar radiation pressure.  $A$  is the exposed area to the sun.

There are two kinds of reflection-specular and diffuse. Now consider solar radiation incident on a specular, reflecting ( $C_R > 1$ ), surface element  $A$ , whose normal makes a solar incidence angle,  $\phi_{inc}$ , with the Sun line.



**Figure (2.2) Incident Solar Radiation. The incident radiation produces one reflected beam plus reflected,  $dF_r$ , and absorbed,  $dF_a$ , forces.**

Where the  $dF_N, dF_T$  are normal and tangential force components [1].

The reflection process is two-fold: absorption based on the reflectivity ( $C_R$ ), followed by specular reflection. Figure (2.2) permits us to find expressions for the forces imparted by absorption and reflection.

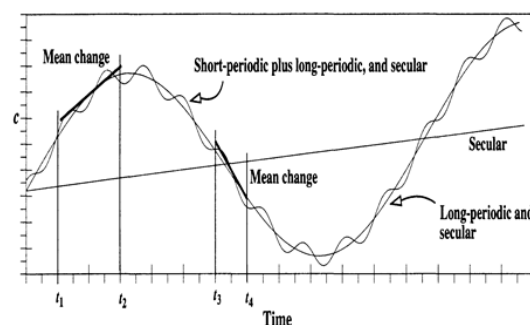
Newton's second law permits us to determine the acceleration due to the direct solar radiation and may be written as:

$$a_R = P_R C_R \frac{A}{m} \quad \dots \dots \dots (2)$$

Where  $\frac{A}{m}$  is the cross section area to the mass ratio of the satellite. Radiation pressure obviously is turned off if the radiation is blocked by an obstacle between the sun and the satellite. The Moon or the Earth may serve as obstacles, this explained in section 4.

### 3. Method of perturbation

Examining the effect of perturbations on the orbital elements, we must characterize how they vary over time. Perturbations on orbital motion result in secular and periodic changes. Secular changes in a particular element vary linearly over time, or in some cases, proportionally to some power of time, such as a quadratic. The important point is that secular terms grow with time, and errors in secular terms produce unbounded error growth variations. Periodic changes are either short- or long periodic, depending on the length of time required for an effect to repeat (short-periodic effects typically repeat on the order of the satellite's period or less while long-periodic effects have cycles considerably longer than one orbital period typically one or two orders of magnitude longer). These long-periodic effects are often seen in the motion of the node and perigee and can last from. Figure (3.1) shows an example of each of these effects. The straight line shows secular effects. The large oscillating line shows the secular plus long-periodic effects, and the small oscillatory line, which combines all three, shows the short-periodic effects.



**Figure (3.1) Effect of perturbation forces on orbital elements.**

The method of perturbations describes a class of mathematical techniques for generating analytical solutions which describe the motion of a satellite subject to disturbing forces. The method of perturbations is that one or more small disturbing forces cause small deviations from the known solution to the unperturbed problem. These small perturbing forces can be associated with small

parameters which characterize the magnitude of the disturbing forces

$$\frac{dc_i}{dt} = \epsilon_1 f_i + \epsilon_1 g_i \quad \dots \dots \dots (3)$$

$$\epsilon_1 \ll 1, \epsilon_2 \ll 1$$

and an approximation to the true solution can be developed as a power series in these small parameters:

$$c = c_0 + \epsilon_1 \alpha_1(t) + \frac{\epsilon_1^2}{2!} \alpha_2(t) + \dots + \epsilon_2 \alpha \beta_1(t) + \frac{\epsilon_2^2}{2!} \beta_2(t) + \dots + \frac{\epsilon_1 \epsilon_2}{2!} \gamma_2(t)$$

Note that the order of the theory is defined as the highest power of the expansion parameter retained in the series expansion. It's common to identify an analytical perturbation theory as "first order," "second order," and so on. Solar flux for solar radiation pressure is an example of small parameters. The variations of perturbations equations of motion are a system of first order differential equations that describe the rates of change for the time-varying elements,

$$\frac{dc}{dt} = f(\vec{c}, t)$$

Using matrix notation, the relationship between the osculating-element rates and the disturbing acceleration gives as [1]

$$\begin{bmatrix} \sum_{i=1}^6 \frac{\partial \vec{x}(\vec{c}, t)}{\partial c_i} \frac{dc_i}{dt} \\ \sum_{i=1}^6 \frac{\partial \vec{x}(\vec{c}, t)}{\partial c_i} \frac{dc_i}{dt} \end{bmatrix} = \begin{bmatrix} 0 \\ \vec{a}_{pert} \end{bmatrix} \dots \dots \dots (4)$$

Gauss's form is advantageous for non conservative forces because it's expressed directly from the disturbing acceleration, Burns derive the time rates of change of orbital elements using elementary dynamics to obtain a general form of the Gaussian VOP equations from equation (4) matrix expressing it as two separate equations and not using the matrix notation [1]:

$$\sum_{i=1}^6 \frac{\partial \vec{x}(\vec{c}, t)}{\partial c_i} \frac{dc_i}{dt} = 0 \dots \dots \dots (5)$$

$$\sum_{i=1}^6 \frac{\partial \vec{x}(\vec{c}, t)}{\partial c_i} \frac{dc_i}{dt} = \vec{a}_{pert} \dots \dots \dots (6)$$

Taking the dot product of the first equation with  $\frac{\partial c_j}{\partial \vec{x}}$

and the second by  $\frac{\partial c_j}{\partial \vec{x}}$  and adding the two relations result in

$$\sum_{i=1}^6 \left( \frac{\partial c_j}{\partial \vec{x}} \cdot \frac{\partial \vec{x}}{\partial c_i} + \frac{\partial c_j}{\partial \vec{x}} \cdot \frac{\partial \vec{x}}{\partial c_i} \right)$$

Because the elements depend only on position and velocity and are mutually independent, the quantity inside the parenthesis reduces to the Kronecker  $\delta$  function,  $\delta_{j,i} = 1$  for  $j = i$  and  $\delta_{j,i} = 0$  for  $j \neq i$ . Therefore, equation (6) becomes

$$\sum_{i=1}^6 \delta_{j,i} \frac{dc_i}{dt} = \frac{\partial c_j}{\partial \vec{x}} \cdot \vec{a}_{pert}$$

For many applications, it's convenient to express the rates of change of the elements explicitly in terms of the disturbing forces, Gauss chose to develop the equations in the RSW system, therefore the Gaussian

form of the variation of parameters equations using the force components resolved in the RSW system, disturbing force becomes [1,10]:

$$\frac{da}{dt} = \frac{2na^3}{\sqrt{1-e^2}} F \left[ eS(v) \sin v + T(v) \frac{p}{r} \right] \dots (7a)$$

$$\frac{de}{dt} = na^2 \sqrt{1-e^2} F \left[ S(v) \sin v + T(v) \left\{ \cos v + \frac{1}{e} \left( 1 - \frac{r}{a} \right) \right\} \right] \dots \dots \dots (7b)$$

$$\frac{di}{dt} = \frac{na^2}{\sqrt{1-e^2}} FW \frac{r}{a} \cos(v + \omega) \dots \dots (7c)$$

$$\sin i \frac{d\Omega}{dt} = \frac{na^2}{\sqrt{1-e^2}} FW \frac{r}{a} \sin(v + \omega) \dots (7d)$$

$$\frac{d\omega}{dt} = -\cos i \frac{d\Omega}{dt} + \frac{na^2 \sqrt{1-e^2}}{e} F \left[ -S(v) \cos v + T(v) \left( 1 + \frac{r}{p} \right) \sin v \right] \dots \dots \dots (7e)$$

$$\frac{dM}{dt} = n - 2na^2 FS(v) \frac{r}{a} - \sqrt{1-e^2} \left[ \frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} \right]$$

Here  $p = a(1-e^2)$ ,  $v$  is the true anomaly;  $n^2 a^3 FS(v)$ ,  $n^2 a^3 FT(v)$ , and  $n^2 a^3 FW$  are three components of the disturbing force due to solar radiation pressure along the satellite's radius vector, perpendicular to it in the orbit plane in the direction of satellite motion, and  $W$  normal to the orbit plane respectively.

Where  $F = \frac{aR}{\mu}$  denotes the magnitude of the radiation-pressure force per unit satellite mass.

Where  $S(v)$ ,  $T(v)$ , and  $W$  given by Kozi [11], and Cook [12] as:

$$\begin{cases} S(v) \\ T(v) \end{cases} = -\cos^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \begin{cases} \cos \\ \sin \end{cases} (\lambda_{\odot} - v - \omega - \Omega) - \sin^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \begin{cases} \cos \\ \sin \end{cases} (\lambda_{\odot} - v - \omega + \Omega) - \frac{1}{2} \sin i \sin \epsilon \begin{cases} \cos \\ \sin \end{cases} ((\lambda_{\odot} - v - \omega) - \begin{cases} \cos \\ \sin \end{cases} (-\lambda_{\odot} - v - \omega)) - \sin^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \begin{cases} \cos \\ \sin \end{cases} (-\lambda_{\odot} - v - \omega + \Omega) - \cos^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \begin{cases} \cos \\ \sin \end{cases} (-\lambda_{\odot} - v - \omega - \Omega) \dots \dots \dots (8a)$$

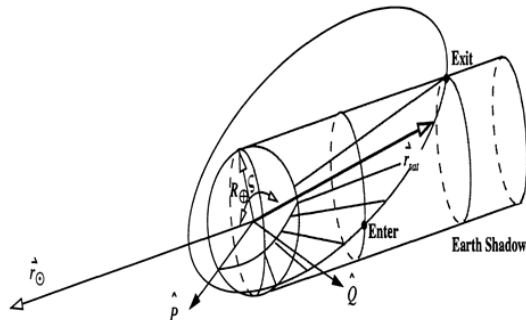
$$W = \sin i \cos^2 \frac{\epsilon}{2} \sin(\lambda_{\odot} - \Omega) - \sin i \sin^2 \frac{\epsilon}{2} \sin(\lambda_{\odot} + \Omega) - \sin i \sin^2 \frac{\epsilon}{2} \sin(\lambda_{\odot} + \Omega) - \cos i \sin \epsilon \sin \lambda_{\odot} \dots \dots (8b)$$

Where  $\epsilon = 23.5^\circ$  the obliquity of the ecliptic, and  $\lambda_{\odot}$ , the ecliptic longitude of the Sun.

#### 4. Earth shadow

The earth's shadow can first modeled as a cylinder, the solar rays supposed to be parallel by [10, 13]; Vokrouhlicky 1994 develop an algorithm to model with great precision the changing solar radiation force exerted on a satellite during the passages from the full sunlight to the shadow. When the satellite is eclipsed it's not exposed to solar radiation pressure, figure (4.1) show the geometry the shadow, the angular separation with the sun is[1]:

$$\cos \zeta = \frac{\vec{r}_{\odot} \cdot \vec{r}_{sat}}{r_{\odot} r_{sat}} \dots \dots \dots (9)$$



**Figure (4.1) Entry-Exit geometry for solar radiation.**

The final form shadow function given by Vallado is

$$s_{shadow} = R_{\odot}^2 (1 + e \cos v)^2 + p^2 (\beta_1 \cos v + \beta_2 \sin v)^2 - p^2 \dots \dots \dots (10)$$

Converting to cosine terms, gives fourth order equation in the cosine of the true anomaly

$$s_{shadow} = \alpha_1 \cos^4 v + \alpha_2 \cos^3 v + \alpha_3 \cos^2 v + \alpha_4 \cos v + \alpha_5 \dots \dots \dots (11)$$

Solving the quadratic equation yield the desired values of true anomaly for entry and exit; if the value of  $s_{shadow}$  in equation (10) is changing from negative to positive the computed value of  $v$  is entry else is represent the exit. Where  $\alpha$ ,  $\alpha_n$  given in appendix A. Kozai [11] obtain the following expressions for the perturbations suffered by a satellite that moves in sunlight; where  $E_2$  and  $E_1$  are eccentric anomaly entering and exiting of the satellite Earth's shadow.

$$\Delta a = 2a^3 F \left[ S \cos E + T(1 - e^2)^{1/2} \sin E \right]_{E_1}^{E_2}$$

$$\Delta e = a^2 F (1 - e^2)^{1/2} \left[ \frac{1}{2} S (1 - e^2)^{1/2} \cos 2E + T \left( \frac{3}{2} E - 2e \sin E + \frac{1}{4} \sin 2E \right) \right]_{E_1}^{E_2}$$

$$\Delta i = a^2 F W (1 - e^2)^{-1/2} \left[ \left[ -\frac{3}{2} e E + (1 - e^2) \sin E - \frac{e}{4} \sin 2E \right] \cos \omega + (1 - e^2)^{1/2} \left( \cos E - \frac{e}{4} \cos 2E \right) \sin \omega \right]_{E_1}^{E_2}$$

$$\sin i \Delta \Omega = a^2 F W (1 - e^2)^{-1/2} \left[ \left[ \frac{3}{2} e E + (1 + e^2) \sin E - \frac{e}{4} \sin 2E \right] \sin \omega - (1 - e^2)^{1/2} \left( \cos E - \frac{e}{4} \cos 2E \right) \cos \omega \right]_{E_1}^{E_2}$$

$$\Delta \omega = -\cos i \Delta \Omega + \frac{a^2 F (1 - e^2)^{1/2}}{e} \left[ S \left( -\frac{3}{2} E + e \sin E + \frac{1}{4} \sin 2E \right) + T (1 - e^2)^{-1/2} \left( e \cos E - \frac{1}{4} \cos 2E \right) \right]_{E_1}^{E_2}$$

$$\Delta M = -(1 - e^2)^{1/2} (\Delta \omega + \cos i \Delta \Omega) - 3a^2 F \left[ S \left[ -\frac{3}{2} e E + \left( \frac{5}{3} + \frac{2}{3} e^2 \right) \sin E - \frac{5}{13} e \sin 2E \right] - T (1 - e^2)^{1/2} \left( \frac{5}{3} \cos E - \frac{5}{12} e \cos 2E \right) - [S \cos E + T (1 - e^2)^{1/2} \sin E] (E - e \sin E) \right]_{E_1}^{E_2}$$

## 5. Result and discussion

In order to illustrate the solar radiation effect on the behavior of orbital element, a computer simulation has been developed using the Matlab R2010a to determine the perturbation effect on orbital elements of satellite under the influence of radiation pressure. Figures ( 5.1 – 5.4 ) shows the perturbation due to solar radiation pressure over four day and figures ( 5.5 – 5.7 ) over one year beginning at MJD 2455194 on orbital elements of US/French altimetry satellite TOPEX / Poseidon has  $a = 7714$  km,  $e = 0.001$  and  $i = 66^\circ$  at two different values of area to mass ratio  $A/m$ ; the blue dashed line for ratio equal to  $0.008 \text{ m}^2/\text{kg}$  and the red line for ratio  $0.02 \text{ m}^2/\text{kg}$ . the results shows that the orbital elements of satellite much more affected by radiation pressure at area mass ratio 0.02 than that ratio equal to 0.008.

Figures (5.8 – 5.11) shows the perturbation on orbital elements of GPS like satellite has semi major axis  $a = 26550$  km,  $e = 0.02$ ,  $i = 55^\circ$ , and figures (5.12 – 5.15) represent the solar radiation perturbation on orbital elements of space debris which has initial semi major axis  $a = 42112$  km,  $e = 0.1$ ,  $i = 5.73^\circ$ ; the results show the perturbation effect increase as the altitude of the satellite increase.

The perigee and apogee high's of satellite  $a = 7714$  km will vary by about  $\pm \Delta e = 0.308$  km; and for satellite  $a = 26550$  km the perigee and apogee high's is change by  $\pm \Delta e = 5.3$  km. Also for the two satellites and the space debris at different altitude and the same  $A/m$  ratio, the satellite with bigger semi-major axis expected to show the larger perturbation due to radiation pressure.

The solar radiation pressure effect is applied on two different orbits which only differ by initial right ascension of ascending node; in the first case the ascending node coincides with the vertical equinox,  $\Omega = 0^\circ$ , while in the second case when the  $\Omega = 180^\circ$ . The results show that the solar radiation pressure on orbital elements is vary by changing the magnitude of node.

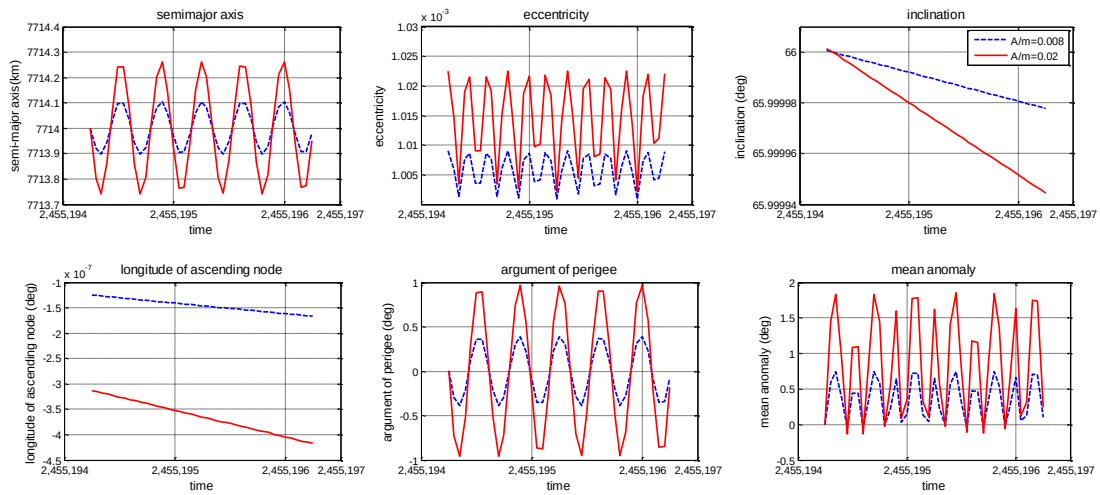


Figure (5.1) the perturbation due to solar radiation pressure in four day at different  $A/m$  ratio. ( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 0$ ).

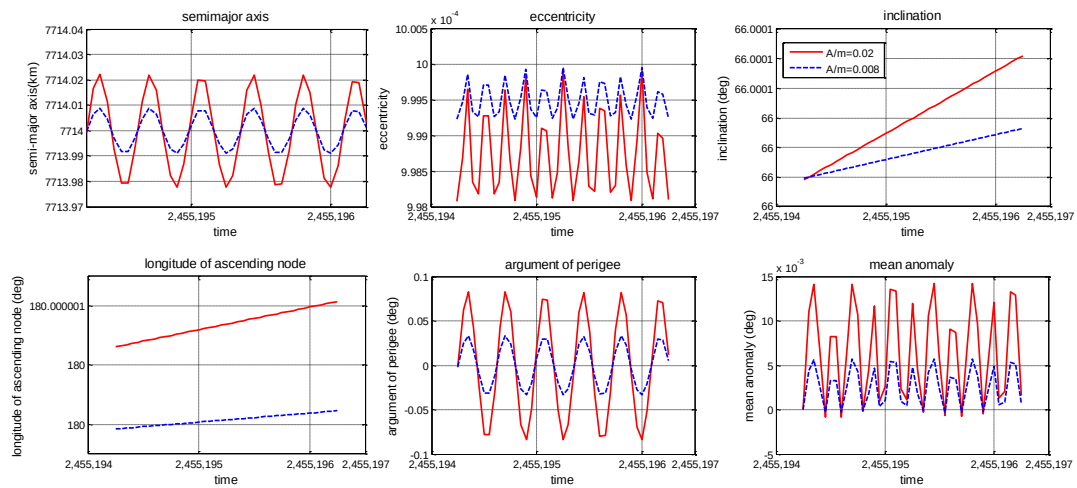


Figure (5.2) the perturbation due to solar radiation pressure in four day at different  $A/m$  ratio. ( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 180$ ).

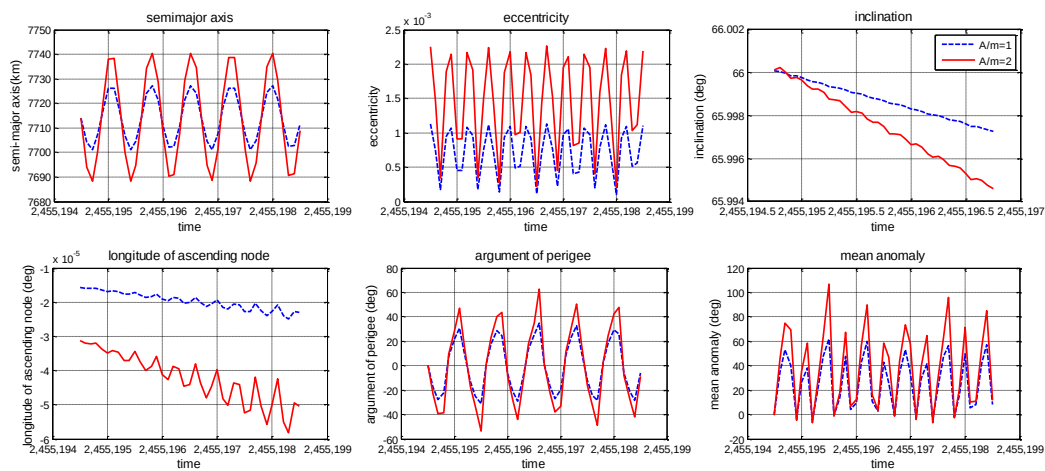


Figure (5.3) the perturbation due to solar radiation pressure in four day at different  $A/m$  ratio. ( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 0$ ).



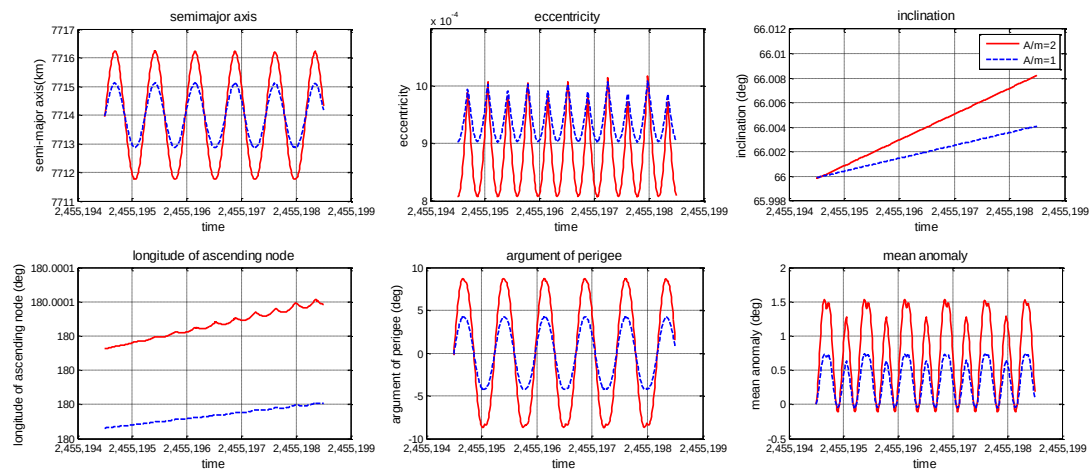


Figure (5.4) the perturbation due to solar radiation pressure in four day at different A/m ratio.  
( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 180$ ).

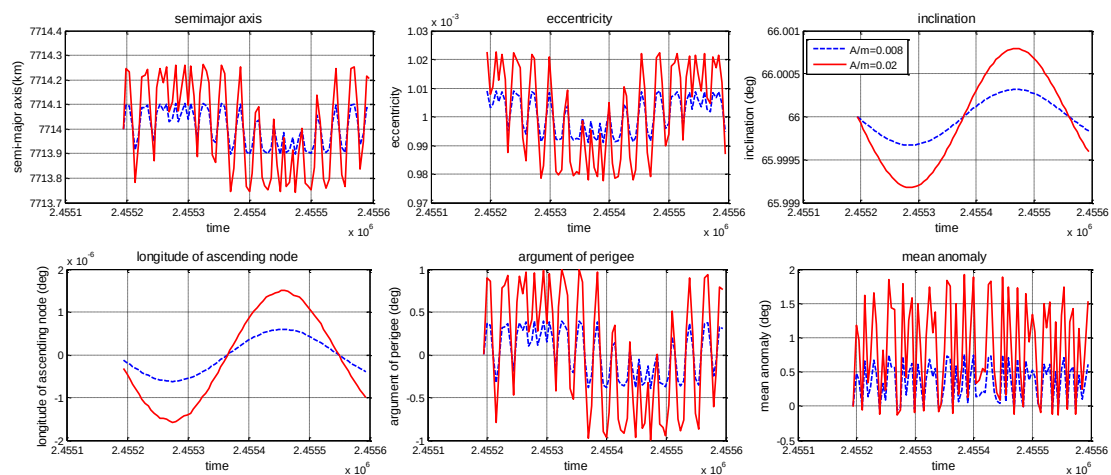


Figure (5.5) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 0$ ).

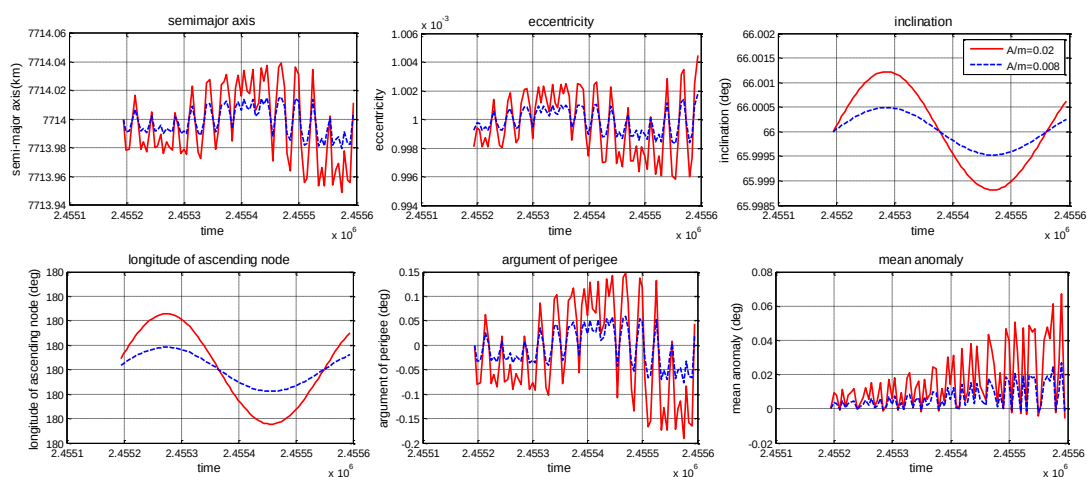
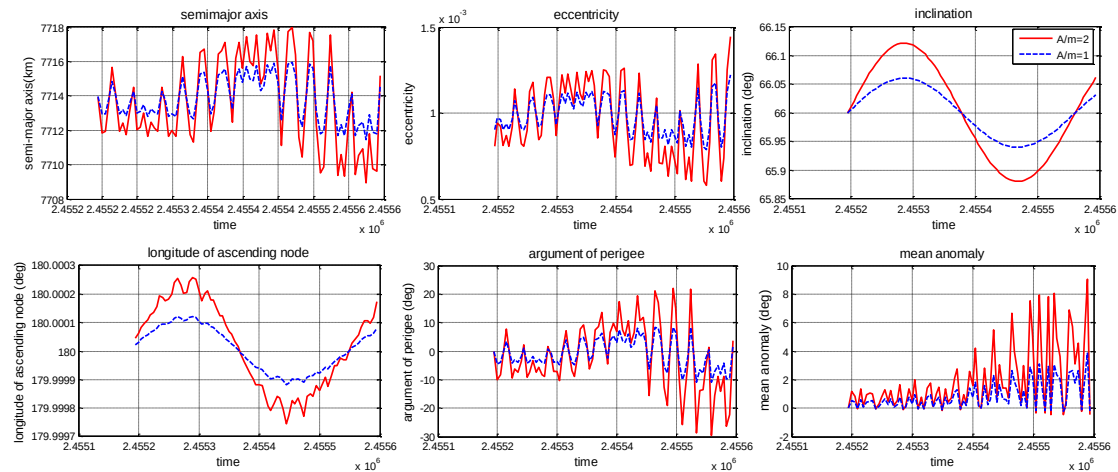
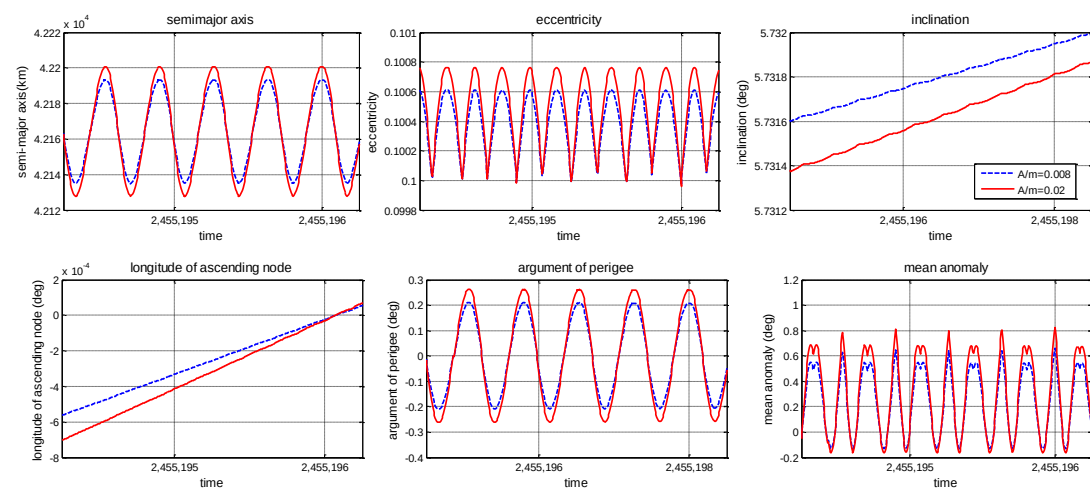


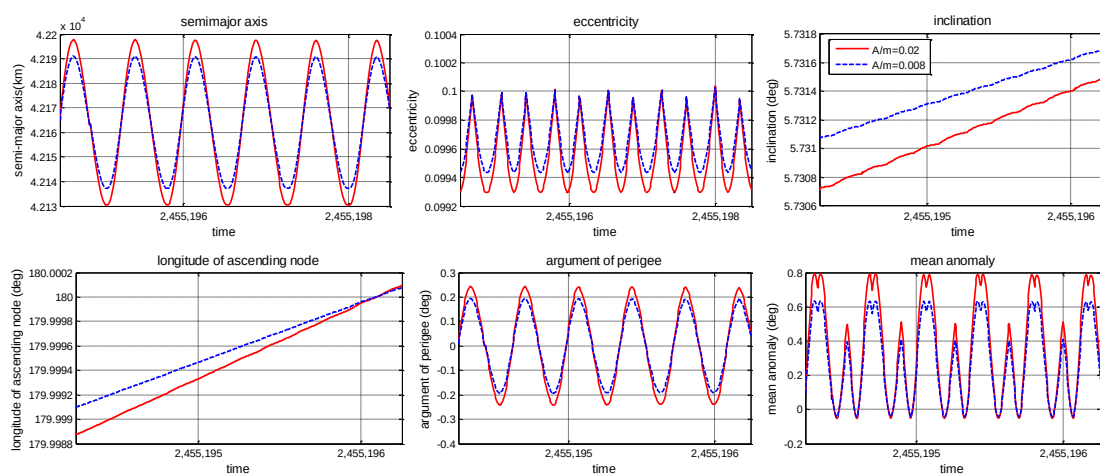
Figure (5.6) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 180$ ).



**Figure (5.7) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 7714\text{km}$ ,  $e = 0.001$ ,  $\Omega = 180$ ).**



**Figure (5.8) the perturbation due to solar radiation pressure in four day at different A/m ratio.  
( $a = 42164\text{km}$ ,  $e = 0.1$ ,  $\Omega = 0$ )**



**Figure (5.9) the perturbation due to solar radiation pressure in four day at different A/m ratio.  
( $a = 42164\text{km}$ ,  $e = 0.1$ ,  $\Omega = 180$ )**

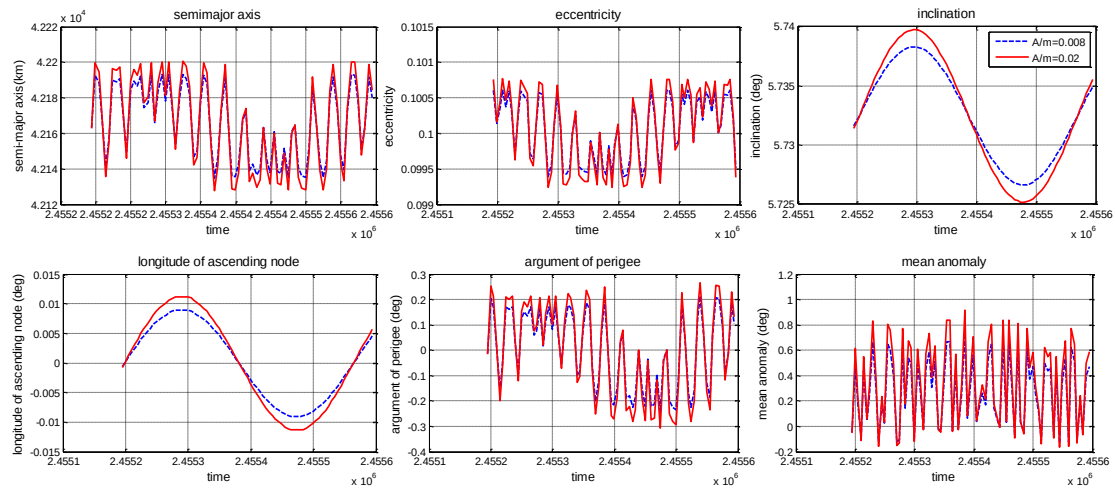


Figure (5.10) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 42164\text{km}$ ,  $e = 0.1$ ,  $\Omega = 0$ )

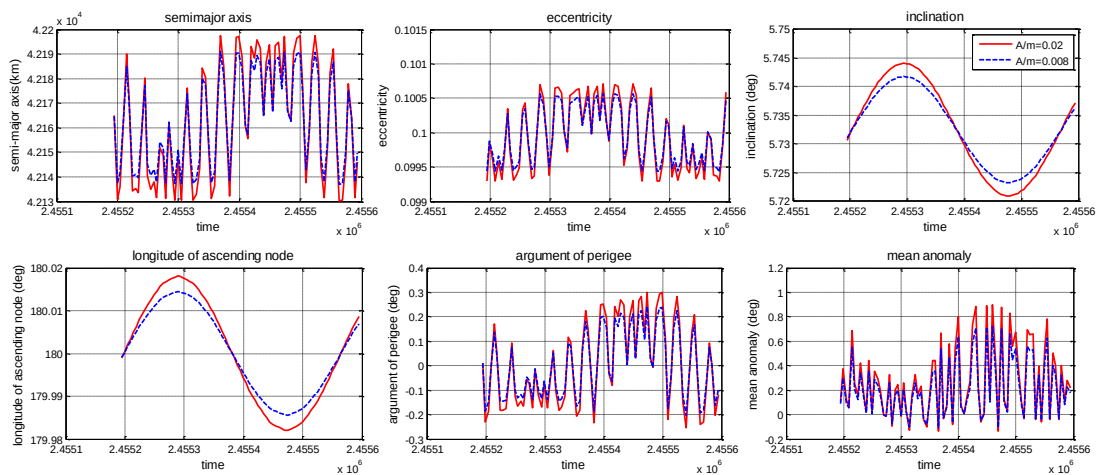


Figure (5.11) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 42164\text{km}$ ,  $e = 0.1$ ,  $\Omega = 180$ )

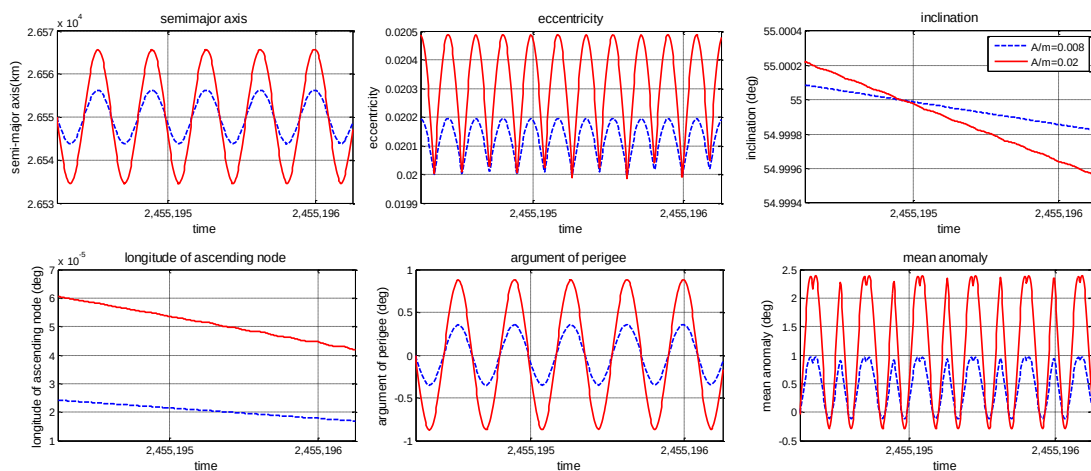


Figure (5.12) the perturbation due to solar radiation pressure in four day at different A/m ratio.  
( $a = 26550\text{km}$ ,  $e = 0.02$ ,  $\Omega = 0$ ).



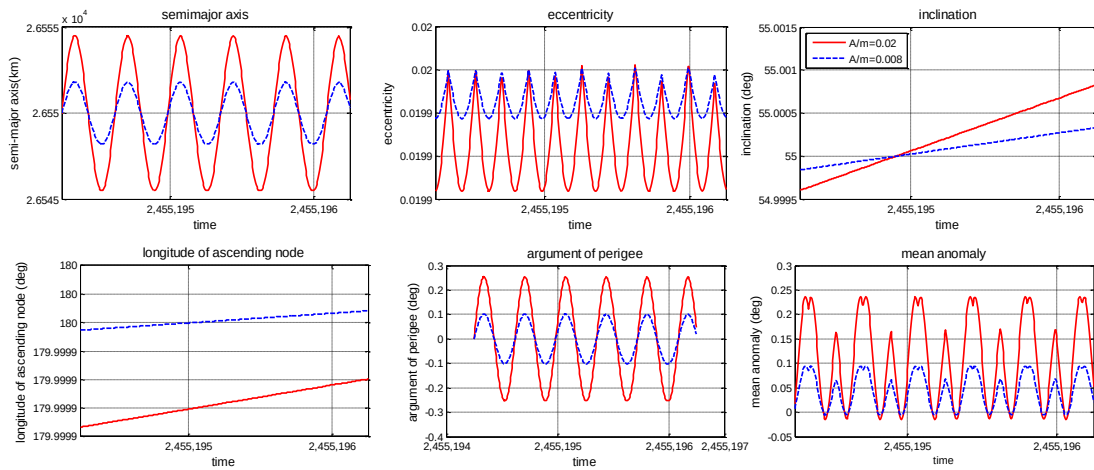


Figure (5.13) the perturbation due to solar radiation pressure in four day at different A/m ratio.  
( $a = 26550\text{km}$ ,  $e = 0.02$ ,  $\Omega = 180$ ).

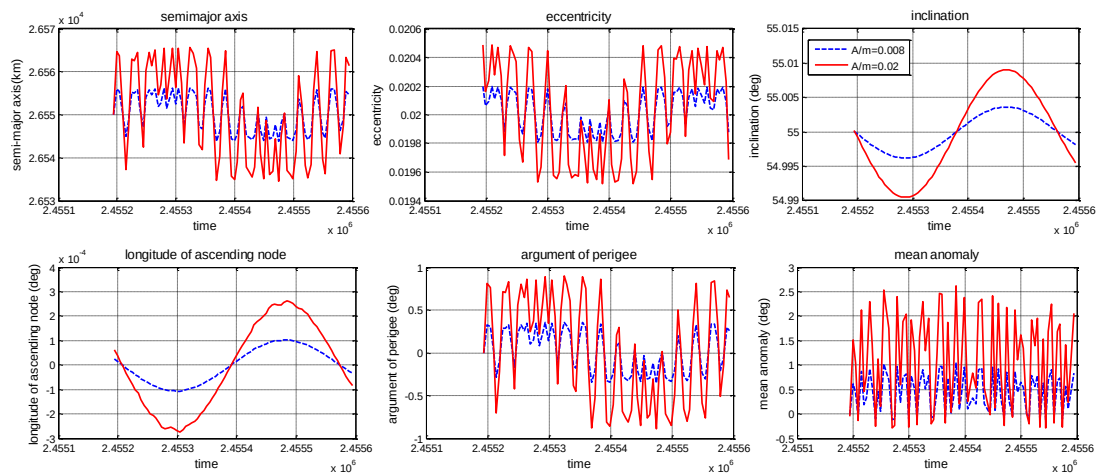


Figure (5.14) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 26550\text{km}$ ,  $e = 0.02$ ,  $\Omega = 0$ ).

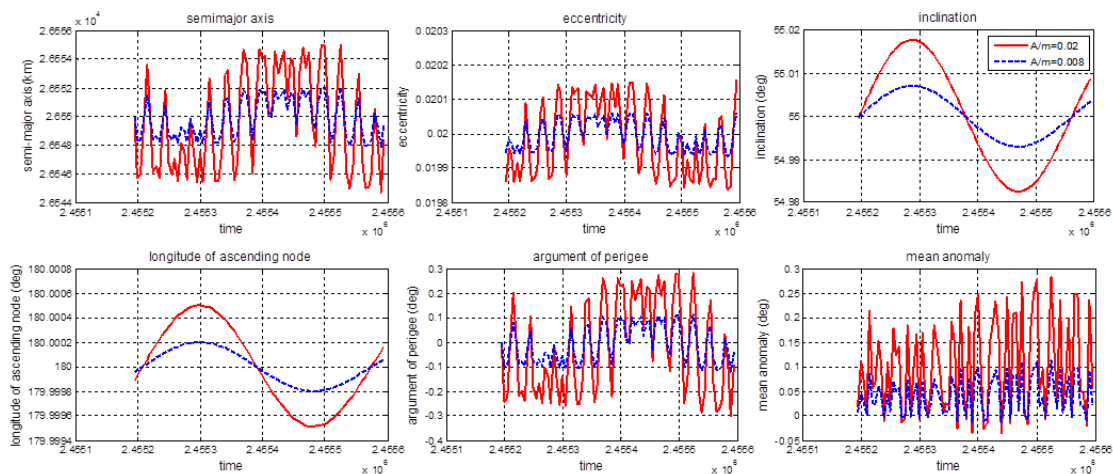


Figure (5.15) the perturbation due to solar radiation pressure in one year at different A/m ratio.  
( $a = 26550\text{km}$ ,  $e = 0.02$ ,  $\Omega = 180$ ).

## 6. Conclusions

In this paper the change in orbital elements are computed due to the solar radiation pressure effect. The variations in the orbital is the largest for the big

value of area to mass ratio, also the effect of the solar radiation is depend on the altitude of satellite and the orientation of the plane of the orbit.

## Appendix

$$\alpha_1 = \alpha^4 e^4 - 2\alpha^2(\beta_2^2 - \beta_1^2)e^2 + (\beta_1^2 + \beta_2^2)^2$$

$$\alpha_2 = 4\alpha^4 e^3 - 4\alpha^2(\beta_2^2 - \beta_1^2)e$$

$$\alpha_3 = 6\alpha^4 e^2 - 2\alpha^2(\beta_2^2 - \beta_1^2) - 2\alpha^2(1 - \beta_2^2)e^2 + 2(\beta_2^2 - \beta_1^2)(1 - \beta_2^2) - 4\beta_2^2\beta_1^2$$

$$\alpha_4 = 4\alpha^4 e - 4\alpha^2(1 - \beta_2^2)e$$

$$\alpha_5 = \alpha^4 - 2\alpha^2(1 - \beta_2^2) + (1 - \beta_2^2)^2$$

$$\alpha = \frac{R_{\oplus}}{p}$$

$$\beta_1 = \frac{\vec{r}_{\odot} \cdot \hat{p}}{r_{\odot}}, \quad \beta_2 = \frac{\vec{r}_{\odot} \cdot \hat{Q}}{r_{\odot}}$$

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## تأثير ضغط الإشعاع الشمسي على العناصر المدارية لمدارات الأقمار الصناعية

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### الملخص:

تناول هذا البحث دراسة تأثير ضغط الإشعاع الشمسي وهو واحد من اهم قوة الاضطرابات الغير المحافظة على العناصر المدارية للأقمار الاصطناعية ذات الارتفاعات المختلفة وعند قيم مختلفة لنسبة المساحة للكتلة وقد استخدم برنامج الماتلاب لاجراء الحسابات. وقد اشارة النتائج المحسوبة ان العناصر المدارية تتأثر بشكل اكبر بالإشعاع الشمسي عند القيم الاكبر لنسبة المساحة للكتلة واشتمل البحث تأثر الاقمار الصناعية عند قيمتين مختلفتين من قيم العقدة الصاعدة.

**الكلمات المفتاحية:** ضغط الإشعاع الشمسي، نسبة المساحة للكتلة، العناصر المدارية، ظل الشمس، الاضطرابات.