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Application Backstepping Method for Stabilizing and Solving Delayed Lotaka-Volterra with A Dynamical Graph

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ABSTRACT

The primary goal of this work is to introduce dynamic graphs. Specifically, it will demonstrate that the matrix is the basic matrix of interconnections (adjacency). It functions to explain a particular graph D's nominal structure based on the presumption that graph D's lines' functional dependence that is, the matrix E's elements (edges) are arranged so, that equation. It can be obtained by two scalar equations and described the evolution of the dynamic matrix E over time.

To transform nonlinear differential equations derived from delay differential equations (DDEs) to linear differential equations, the purpose of using a dynamical graph. With this method, we applied on the biological problem of Lotaka-Volterra delay to studying stability by the backstepping method to delay differential equation (DDE) system to investigate stability on the impact of unsure interconnections between subsystems and solve it.

Keywords: Delay Differential Equations (DDEs), Dynamical graph, Backstepping, Method of steps

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تطبيق طريقة التغذية الراجعة مع الرسم البياني الديناميكي لتثبيت وحل معادلة تفاضلية تباطؤية

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الملخص

الهدف الرئيسي من هذه البحث هو اعتماد طريقة التغذية الراجعة لنظام معادلات التأخير التفاضلية مع الرسم البياني الديناميكي من خلال اعتماد الدالة بين خطوط (الحافات) الرسم البياني D هو مثل في النظام المعادلات التفاضلية العادية (ODEs)، الذي يصف التطور في وقت المصفوفة الديناميكية، سيتم إعطاؤه معادلتين عدديتين كنظام معمم للرسم البيانية ذات الخطوط (الحواف). بعد ذلك، سوف ندرس استقرارية وحل لمشكلة البيولوجية في معادلة التفاضلية التباطؤية (DDE)

INTRODUCTION

Backstepping is the method used to design stabilizing controllers. for a specific type of nonlinear dynamical systems that Petar V. Kokotovic and others evolved around 1990. The components of these systems emanate via an irreducible subsystem that may be stabilized through different methods. Starting with the known-stable system. The designer can start the design process and because of the recursive structure forces additional controllers to be backed out gradually. Stabilizing each outer subsystem. The process is completed after the final external control is reached. Consequently, it works in a recursive manner step by step, this process is known as backstepping. A class for studding non-linear systems of ODEs may now make advantage of generalized backstepping design. A model is provided in ⁽¹⁾ to analysis prey-predator systems and will be stabilized in this paper by introducing a generalization methodology built upon the DDE's backstepping control technique. By employing the method of steps, this technique primarily converts the delay differential equations (DDEs) with retarded arguments into an ordinary differential equation (ODE) system. In addition, compared to other established procedures. This one

is more effective and less complex. The second Lyapunov stability approach served as the foundation for the backstepping control scheme's basic nonlinear control methodology. Because of its recursive and methodical design process for nonlinear feedback control, it has attracted a lot of interest. This method can prevent the cancellation of undesirable terms and provides a variety of design options for accommodating nonlinearities. Another significant benefit of this approach is that it offers a systematic, step-by-step algorithm-based process for designing stabilizing controllers. With this, a systematic strategy is used for creating feedback control laws and Lyapunov functions^(2, 3). Understanding the stability properties helps assess the susceptibility of periodic solutions to small perturbations ⁽⁴⁾.

A basic graph G is composed of two sets of elements. The vertex (or node) set $V(G)$ is nonempty, as well as distinct unordered pairs of the vertices $E(G)$. Vertices v and u are said to be joined by an edge denoted by the symbol vu . When an edge vu connects two vertices v and u of a graph G and those vertices v and u are incident with that edge, we say that the two vertices v and u are

adjacent. A vertex in common between two distinct edges e and f makes them adjacent.

Moreover, with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{e_1, e_2, \dots, e_m\}$ edge set in a basic graph G a basic graph G can be represented using matrices. The $n \times n$ adjacency matrix $(G) = [a_{ij}]$ is one such matrix, where

$$a_{ij} = \begin{cases} 1, & \text{if } e \in E(G) \\ 1, & \text{if } e \notin E(G) \end{cases} \text{ if } e = v_i v_j.$$

Graph G 's adjacency matrix has zero entries along the main diagonal and is symmetric [0,1] (5-7).

1. Application backstepping method with a dynamical graph to solve a system of DDEs

To stabilize and solve the biological problem in the delayed Lotka-Volterra in the following natural way which given by (8): nonlinear independent Delay Lotka Volter to linear DDE

$$\begin{cases} \dot{x}_1(t) = a + bx_1(t-\tau) - cx_2(t)x_1(t) \\ \dot{x}_2(t) = e + cx_1(t)x_2(t) - dx_2(t) \end{cases} \dots (1)$$

When $a = 1$, $b = 4$, $c = 0.05$, $d = 0.01$, $e = 2$ and $\tau = 5$

the given system (1) becomes after disjunction on the control functions u_1 and u_2 using the generalized backstepping technique as follow:

$$\begin{cases} \dot{x}_1(t) = a + bx_1(t-\tau) - cx_2(t)x_1(t) + u_1(t) \\ \dot{x}_2(t) = e + cx_1(t)x_2(t) - dx_2(t) + u_2(t) \end{cases} \dots (2)$$

With initial condition $x_1(t) = \varphi_{10}(t) = 2t + 0.5$, $\forall -1 \leq t \leq 0$

The solution for the initial time-step interval [0,1] can be acquired by employing the subsequent technique:

$$\begin{cases} \dot{x}_1(t) = 1 + 4\varphi_{10}(t-5) - 0.05x_2(t)x_1(t) \\ \dot{x}_2(t) = 2 + 0.05x_1(t)x_2(t) - 0.01x_2(t) \end{cases} \dots (3)$$



and then by using the initial conditions, we obtain:

$$\begin{cases} \dot{x}_1(t) = 1 + 4(2(t-5) + 0.5) - 0.05x_2(t)x_1(t) \\ \dot{x}_2(t) = 2 + 0.05x_1(t)x_2(t) - 0.01x_2(t) \end{cases} \dots (4)$$

Thus, the outcomes of the nonlinear ODEs are as follows:

$$\begin{cases} \dot{x}_1(t) = -37 + 8t - 0.05x_2(t)x_1(t) \\ \dot{x}_2(t) = 2 + 0.05x_1(t)x_2(t) - 0.01x_2(t) \end{cases} \dots (5)$$

Therefor, two scalar equations, defines the dynamic matrix E 's evolution over time.

$$E: \begin{cases} \dot{x}_1(t) = -37 + 8t - 0.05x_2(t)x_1(t) \\ \dot{x}_2(t) = 2 + 0.05x_1(t)x_2(t) - 0.01x_2(t) \end{cases} \dots (6)$$

suppose that x_1 and x_2 are the vertices of a graph and there is a functional dependence between the dynamic graph D 's lines, which correspond to elements x_1 and x_2 of the adjacency matrix, is such that the system of ODE'S (for more details see (7,9). This can be used as a model for particle concentration: Particle concentrations of species X_1 and X_2 are represented by x_1 and x_2 , respectively. Although the interactions between graph edges can be randomly determined, when choosing the concentration of particles model, We wish to investigate whether relationships between a graph's edges could be analogous to relationships between species in a particle concentration.

In order to generalize system (6) for graphs with M lines (edges), we must first determine which model to use:

$$E_1: \dot{x}_i = B + P x_{ij}(t) + R x_i(t) \dots (7)$$

where $x(t) \in R^2$ represents the 2×2 matrixe of the dynamic matrix E_1 , $P = (p_{ij})$, is constant a 2×2 matrix with the proper dimensions, and B is a constant vector make up the system matrices s.t

$$B = \begin{bmatrix} -37 + 8t \\ 2 \end{bmatrix}, \quad R x(t) = \begin{bmatrix} 0 & 0 \\ 0 & -0.01 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

set $x_2(t)x_1(t) = x_{21}(t) = x_1(t)$, $x_1(t)x_2(t) = x_{12}(t) = x_2(t)$ consequently, this indicates that the edge x_{21} denotes the interaction between x_1 and x_2 . Similarly, x_{21} denoted the edge outward from x_1 and is incident on x_2 . Now

$$P x(t) = \begin{bmatrix} 0 & -0.05 \\ 0.05 & 0 \end{bmatrix} \begin{bmatrix} x_2(t)x_1(t) \\ x_1(t)x_2(t) \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & -0.05 \\ 0.05 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

To put a matrix differential equation in standard vector form that expresses $\mathbf{E}_1$, We vectorize $P = (p_{ij})$, which is a $N \times N$ matrix by stacking the rows of P to create a vector differential equation.

$$\mathbf{E}_2: \dot{x} = g(t, x) \quad \dots (8)$$

We begin by converting E to a binary matrix $\bar{P} = (\bar{p}_{ij}) \in \mathbb{R}^{N \times N}$, by applying the following rule:

$$\bar{p}_{ij} = \begin{cases} 1, & \text{if } p_{ij} \neq 0 \\ 0, & \text{if } p_{ij} = 0 \end{cases} \quad \dots (9)$$

in order to demonstrate how the matrix $x(t) \in \mathbb{R}^2$. After many steps which have been illustrated in (5)

$$\text{we get } P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

The following steps will solve and stabilize the resulting system of differential equations (7) using the backstepping approach.

Step 1: Set $z(t) = x(t)$ and examine the stability of the system's first equation (7). As a result, differentiability $z(t)$ with regard to time gives:

$$\dot{z}(t) = \dot{x}(t) = B + Pz(t) + u(t)$$

s.t

$$\begin{aligned} \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} &= \begin{bmatrix} -37 + 8t \\ 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \\ &\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\ \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} &= \begin{bmatrix} -37 + 8t \\ 2 \end{bmatrix} + \begin{bmatrix} z_2(t) \\ z_1(t) \end{bmatrix} + \begin{bmatrix} 0 \\ z_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\ \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} &= \begin{bmatrix} -37 + 8t + z_2(t) + u_1 \\ 2 + z_1(t) - z_2(t) + u_2 \end{bmatrix} \\ \left. \begin{aligned} \dot{z}_1(t) &= -37 + 8t + z_2(t) + u_1 \\ \dot{z}_2(t) &= 2 + z_1(t) - z_2(t) + u_2 \end{aligned} \right\} \quad \dots (10) \end{aligned}$$

Take consider a quadratic form definition of the Lyapunov function, with $x_2(t)$ acting as the virtual controller, as follow

$$V_1(z) = \frac{1}{2} Z^2(t)$$

Consequently, $V_1(z)$'s time derivative becomes:

$$\begin{aligned} \dot{V}_1(z) &= \frac{\partial V_1}{\partial t} = \frac{\partial V_1}{\partial z} \frac{\partial z}{\partial t} \\ &= z(t) \dot{z}(t) \\ &= z_1(t) (-37 + 8t + z_2(t) + u_1) \\ &= z_1(t) (-37 + 8t - z_2(t) + z_1(t) - z_1(t) + u_1) \end{aligned}$$

Suppose that $x_2(t) = \alpha_1(z_1)$ is the controller. If $\alpha_1(z_1) = 0$ and $u_1(t) = 37 - 8t - z_2(t) - z_1(t)$, then

$$\dot{V}_1(z) = -z_1^2(t)$$

It, is a negative definite function, $\forall t \in [0, 1]$, It is a definite negative function for every t in $[0, 1]$. Thus, the first equation of system (10) becomes asymptotically stable due to $\alpha_1(z_1)$ and $u(t)$ are under the recursive feedback control. whenever x_2 is considered as a controller, then $\alpha_1(z_1)$ is a function of estimation.

Step 2: $z_2(t) = x_2(t) - \alpha_1(z_1)$ is the definition of the error between $x_2(t)$ and $\alpha_1(z_1)$, where $\alpha_1(z_1) = 0$ from first step. Then:

$$\dot{z}_2(t) = 2 + z_1(t) - z_2(t) + u_2$$

The second control Lyapunov function is considered like this:

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2(t)$$

Hence, the derivative of V_2 is:

$$\begin{aligned} \dot{V}_2(z_1, z_2) &= \dot{V}_1(z_1) + z_2(t) \dot{z}_2(t) \\ &= -z_1^2(t) + z_2(t)(2 + z_1(t) - z_2(t) + u_2) \end{aligned}$$

If $u_2(t) = -2 - z_1(t) - z_2(t)$, then:

$$\dot{V}_2(z_1, z_2) = -z_1^2(t) - z_2^2(t) \quad \text{for any } t \in [0, 1],$$

likewise a negative definite function.

Consequently, the second equation of system (10) is asymptotically stabilized by the recursive feedback control $u_2(t)$.

As a result, the following feedback controls are achieved for the first time step:

$$\begin{aligned} u_1(t) &= 37 - 8t - z_2(t) - z_1(t) \\ u_2(t) &= -2 - z_1(t) - 2z_2(t), \quad \dots (11) \end{aligned}$$

Therefore, after re-substituting u_1 and u_2 from equations (10) the resulting system of non-constant coefficient ODEs is produced.

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) \\ \dot{x}_2(t) &= -x_2(t) \quad \dots (12) \end{aligned}$$

With initial conditions $x_1(0) = 0.5$, $x_2(t) = 0.25$

Which has the solution

$$\begin{aligned} x_1(t) &= 0.5e^{-t}, \\ x_2(t) &= 0.25e^{-t} \quad \dots (13) \end{aligned}$$

Figure (1) depict the $x_1(t)$ and $x_2(t)$ solutions throughout the first $[0,1]$ time step interval. Figure (2) shown the control function $u_1(t)$ and $u_2(t)$

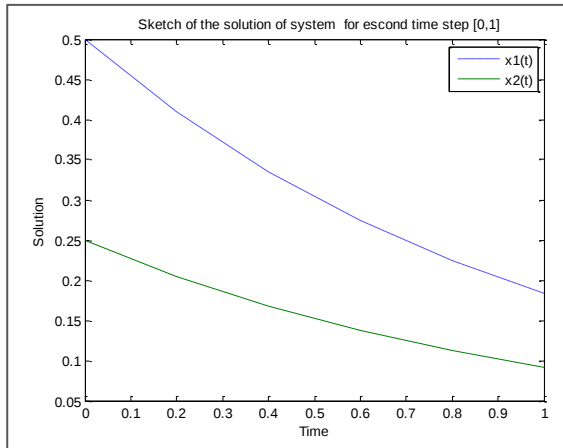


Fig. 1: System (10) solution throughout the time step $[0,1]$

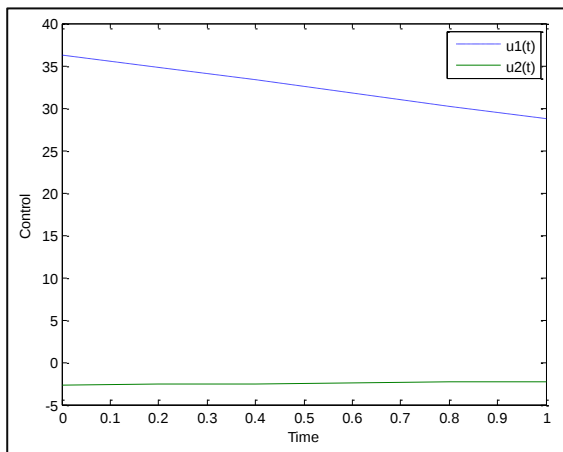


Fig. 2: System (10)'s control function during the time step $[0,1]$

The following solutions produced throughout $[0,1]$ are regarded as forming the second time step interval's initial conditions (8):

$$x_1(t) = \varphi_{11}(t) = 0.5e^{-t}, \quad x_2(t) = \varphi_{21}(t) = 0.25e^{-t} \quad t \in [0,1]$$

Since the outcome in the first step is found through the resolving of the succeeding equations systems following executing the method of steps, in the same way, the solution in the second time step interval $[2,4]$ can be ascertained.

$$\begin{aligned} \dot{x}_1(t) &= 1 + 4\varphi_{11}(t-5) - 0.05 x_1(t) x_2(t) + u_1(t) \\ \dot{x}_2(t) &= 2 + 0.05 x_1(t) x_2(t) - 0.01 x_2(t) + u_2(t) \\ \dots (14) \end{aligned}$$

Therefore, the result of the nonlinear ODEs is given by:

$$\begin{aligned} \dot{x}_1(t) &= 1 + 4(0.5e^{-t+5}) - 0.05 x_1(t) x_2(t) + u_1(t) \\ \dot{x}_2(t) &= 2 + 0.05 x_1(t) x_2(t) - 0.01 x_2(t) + u_2(t) \end{aligned} \quad \dots (15)$$

Similar to the previous step, and according to our presumption, the graph D's lines' functional dependence, which match the matrix 's elements x_1 and x_2 , are such that the system (15). So, according to the scalar equations, depicts how the dynamic matrix E develops over time

$$\begin{aligned} \mathbf{E}: \quad \dot{x}_1(t) &= 1 + 4(0.5e^{-t+5}) - 0.05 x_1(t) x_2(t) + u_1(t) \\ \dot{x}_2(t) &= 2 + 0.05 x_1(t) x_2(t) - 0.01 x_2(t) + u_2(t) \end{aligned} \quad \dots (16)$$

At this point, we wish to generalize system (16) while choosing the mode for graphs having M edges or lines

$$\mathbf{E}_1: \quad \dot{x} = B + Px(t) \quad \dots (17)$$

set $x_2(t)x_1(t) = x_1(t)$, $x_1(t)x_2(t) = x_2(t)$ s.t

$$B = \begin{bmatrix} 1 + 4(0.5e^{-t+5}) \\ 2 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & -0.05 \\ 0.05 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -0.01 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

which make up the system matrices, the standard vector obtained by converting a matrix the differential equation representing $\mathbf{E}_1$ in the previously described model, we get

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 1 + 4(0.5e^{-t+5}) \\ 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -0.01 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \dots (18) \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 + 2e^{-t+5} \\ 2 \end{bmatrix} + \begin{bmatrix} x_2(t) \\ x_1(t) \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 + 2e^{-t+5} + x_2(t) + u_1(t) \\ 2 + x_1(t) + x_2(t) + u_2(t) \end{bmatrix}$$

$$\begin{aligned} \dot{x}_1(t) &= 1 + 2e^{-t+5} + x_2(t) + u_1(t) \\ \dot{x}_2(t) &= 2 + x_1(t) + x_2(t) + u_2(t) \end{aligned} \quad \dots (19)$$

The differential equations system that results is (19). This can be solved and stabilized by applying the backstepping approach, as shown in the subsequent steps

Step 1: Let $z_1(t) = x_1(t)$ Subsequently, time derivative is

$\dot{z}_1(t) = \dot{x}_1(t) = 1 + 2e^{-t+5} - x_2(t) + u_1(t)$
the quadratic Lyapunov function is defined as follows, with $x_2(t)$ treated as a virtual controller:

$$V_1(z_1) = \frac{1}{2} z_1^2(t)$$

Thus, the derivative turns into

$$\begin{aligned} \dot{V}_1(z_1) &= z_1(t) \dot{z}_1(t) \\ &= z_1(t)(1 + 2e^{-t+5} + x_2(t) + u_1(t)) \\ &= z_1(t)(1 + 2e^{-t+5} + x_2(t) + u_1(t) - z_1(t) + z_1(t)) \end{aligned}$$

Assume that the controller $x_2(t) = \alpha_1(z_1)$ and if $u_1(t) = -1 - 2e^{-t+5} - x_2(t) - z_1(t)$, then:

$$\dot{V}_1(z_1) = -z_1^2(t) \quad \text{for every } t \text{ in } t \in \text{Error! Bookmark not defined.}$$

This function is negative definite. Thus, the first equation of the system (19) is asymptotically stabilized by the recursive feedback control $u_1(t)$ and $\alpha_1(z_1)$, when x_2 is taken into account as a controller, α_1 represents an estimation function.

Step2: The definition of the error between $\alpha_1(z_1)$ and $x_2(t)$ is

$z_2(t) = x_2(t) - \alpha_1(z_1)$, where $\alpha_1(z_1) = 0$ from first step. Afterward:

$$\dot{z}_2(t) = \dot{x}_2(t) = 2 + z_1(t) + z_2(t) + u_2(t)$$

The following function is taken into consideration for the second control Lyapunov function:

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2(t)$$

Hence, the derivative of V_2 is:

$$\begin{aligned} \dot{V}_2(z_1, z_2) &= \dot{V}_1(z_1) + z_2(t) \dot{z}_2(t) \\ &= -z_1^2(t) + z_2(t)(2 + z_1(t) + z_2(t) + u_2(t)) \end{aligned}$$

If $u_2(t) = -2 - z_1(t) - 2z_2(t)$, then:

$$\dot{V}_2(z_1, z_2) = -z_1^2(t) - z_2^2(t) \quad \text{for any } t \in [1, 2].$$

Likewise, is a negative definite function. Consequently, the second equation of system (19) is asymptotically stabilized by the recursive feedback control $u_2(t)$.

As a result, the following feedback controls are achieved for the second time step interval [2,4].

$$u_1(t) = -1 - 2e^{-t+5} - x_2(t) - z_1(t),$$

$$u_2(t) = -2 + z_1(t) - 2z_2(t), \quad \dots (20)$$

$u_1(t)$ and $u_2(t)$ are substituted back in system (19). The ensuing system of constant coefficient ODEs is produced:

$$\begin{cases} \dot{x}_1(t) = -x_1(t) \\ \dot{x}_2(t) = -x_2(t) \end{cases} \dots (21)$$

with $x(1) = 0.18394$, $x(1) = 0.091970$ as the initial conditions

which has the solution

$$\begin{aligned} x_1(t) &= 0.5e^{-t}, \\ x_2(t) &= 0.25e^{-t} \quad \dots (22) \end{aligned}$$

Figure (3) depict the $x_1(t)$ and $x_2(t)$ solutions I over the first-time step interval [1,2] with the control function $u_1(t)$ and $u_2(t)$ which given in Figure (4).

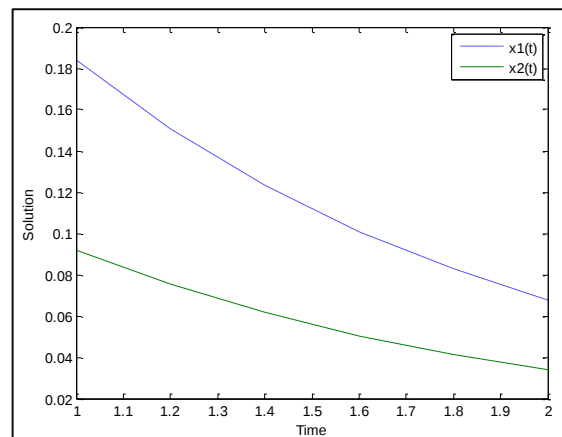


Fig. 3: Solution of system (19) over the time step (1, 10)

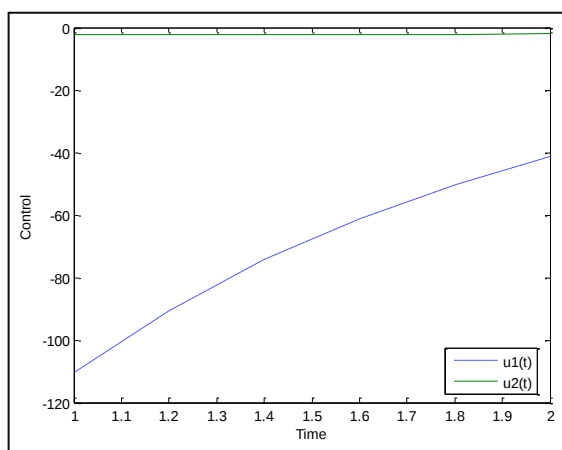


Fig. 4: The control function of system (19) over the time step (1, 10)

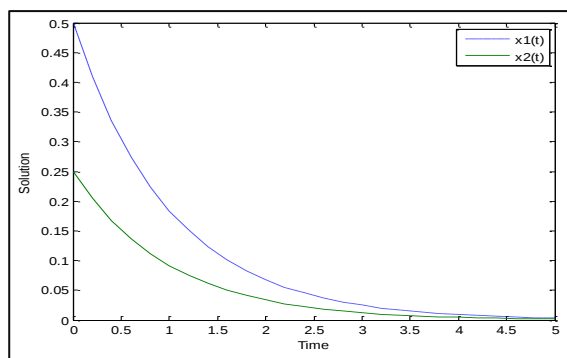


Fig. 5: System (14) solution throughout the time step [0,5]

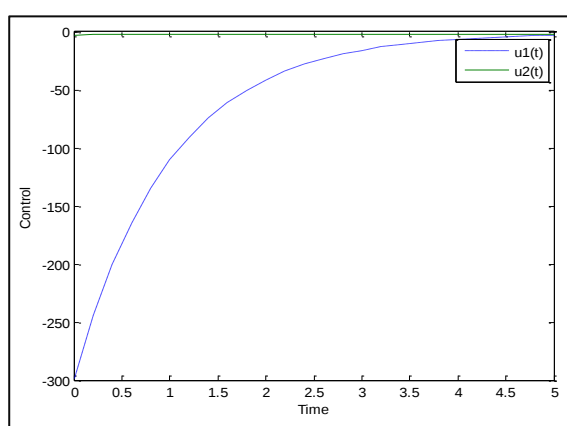


Fig. 6: System (14)'s control function throughout the time step [0,5]

carrying out the same approach, we may find the solutions over the time interval [0,5] which are illustrated in Fig. 5 with the control function Fig. 6.

CONCLUSION

We study the backstepping method for system of DDEs with dynamical graph to transform the nonlinear ODEs outcome from DDEs into linear ODEs. Through applying backstepping method with dynamical graph to stabilized and solved biological problem

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