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A comparison between Bayes estimation and the estimation of the minimal unbiased quadratic Standard of the bi-division variance analysis model in the presence of interaction

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ABSTRACT

In this study, the variance compounds parameters of the mixed bi-division variance analysis sample are estimated. This estimation is obtained, by Bayes quadratic unbiased estimator. The second way to estimate variance compounds parameters of a suggested tow-way analysis of variance mixed model with interaction. estimation is done out by the approach called (MINQUÉ). The estimation approach is conducted on true obtained from departments at the college of agriculture/university of Mosul. These data represent the development of growing various kinds of tomato so that the development represents three factors: the first is tomato kind, this is the first factor (H) and the factor of natural fertilizer rate, and this is the second factor (M), and the interaction between the two factors (HM). A random sample is taken from these data in order to get the random linear sample. The elementary values estimated by Bayes unbiased estimator are very much close to those estimated by variance analysis style when compared with the estimated values of the variance estimation parameters done by minimum standard quadratic unbiased estimation. The elementary values represent random linear sample parameters used to estimate minimum quadratic unbiased standard. The elementary values of the estimations are also obtained via analyzing bi-division variance, then these estimations are employed in estimating minimum quadratic unbiased standard. the estimation results by Bayes approach are very similar to those done by variance analysis.

Aims of the Study

In this study, the variance compounds parameters of the mixed bi-division variance analysis sample are estimated This estimation is obtained, by Bayes quadratic unbiased estimator. The second way to estimate variance compounds parameters of a suggested tow-way analysis of variance mixed model with interaction. estimation is performed out by the approach called (MINQUÉ). the estimation results by Bayes approach are very similar to those done by variance analysis.

1 - Introduction

Variance estimation was suggested by the two astronomers [1]& [2] It is still used in the applications of so many researchers when analyzing so many data. Studies, in this sphere, are always developing. The studies which presented br are very significant the professors at the Mosul university was very significant in this sphere specialized in based on variance compounds of random effects, [3] and [4].From previous studies by researchers, have got the Bayes quadratic unbiased estimations of the bi-division linear samples after assuming the availability of primary information about parameters which contain possible distribution of these parameters. The style of Bayes estimator is easily applied of the general linear sample, where it depends on linear function of variance compounds. It depends, in application, on the first and second factors of possible distributions which consider variance compounds [5] .The linear sample can be represented as follows:

$y = xb + s_1\tau_1 + s_2\tau_2 + s_3\tau_3 + \cdots + s_r\tau_r$ ..(1)

where:

y: represents integer with (n) number of integers.

x: matrix with rank.

τ_r: integer of random variables.

s_r: information matrixes.

b:point of unknown regression parameters.

where:

$e(\tau_i) = 0$, $i = 1,2,\dots,r$

$var(\tau_i) = \sigma_i^2$

$cov(\tau_i, \tau_j) = 0$, $i = j$

$$\left.\begin{array}{l} s = (s_1, s_2, \dots, s_r) \\ \tau = (\tau_1, \tau_2, \dots, \tau_r) \end{array}\right] \text{.. (2)}$$

Sample (2) can be written from (1) as follows:

$y = xb + s\tau$ … (3)

From equation (1) we get: $E(y) = xb$

$var(y) = \sigma_1^2 u_1 + \sigma_2^2 u_2 + \sigma_r^2 u_r$ …(4)

Where these parameters $\sigma_1^2, \sigma_2^2, \dots, \sigma_r^2$ are to be estimated from the data obtained and which are called variance compounds. The known approach used to estimate variance compounds in so many researches is establishing quadratic shapes via this formula

$\acute{Y}\tau_i Y$, i= 1,2, … r

and a solution for the system of equations was reached by:

$\epsilon(\acute{Y}\tau_i Y) = \acute{Y}\tau_i Y$, $i = 1,2,\dots,r$ … (5)

Where the left side of equation (5) is a function by the parameters and get these shapes from rows of factor effects represented by square average [6].The form in equation number (5) faces some difficulties for the estimation was not the minimum fault concerning variances between the estimated values and reality, so scientists have suggested an approach which they named (MINQE), [7]Researchers and scientists have been able to develop this approach and have estimated variance compounds by Bayes method, [8]and [9].

2 -Bayes quadratic unbiased estimator:

The linear sample is represented by the equation:

$$y_{ijk} = \mu + a_i + b_i + c_{ij} + e_{ijk} \text{ …(6)}$$

Where :

k=1,2,3,…,f j=1,2,3,… L i=1,2,3,…,n

(n) represents the number of the levels of the first factor, (i) represents the levels of the second factor, and (f) represents a number of frequencies for the harmonics of factors one and two. assume that a_i , b_i , c_{ij} and e_{ijk} are random independent variables distributed normaly by zero expectation and variance of $\sigma_1^2, \sigma_{2,}^2\sigma_{,3}^2, \sigma_4^2$ and they are called variance compounds as we have already mentioned [10].

Where :

$var(y_{ijk}) = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2$

The form number (6) can be written by matrix as in:

$$\left.\begin{array}{l} Y = F\mu + DU + e \\ e = D_1 a + D_2 b + D_3\gamma + D_4 e \end{array}\right\} \text{… (7)}$$

Y: integer of observations where $N=nlf$

µ: general medium.

F: integer of units.

D_i: design matrix.

U: integer of random effects.

e: integer of random faults.

That the basic is supposed to form (7)

$e \sim N(0, \sigma_e^2 I_N)$

$U \sim N\left(0, DIAG(\sigma_1^2 Iq_1, \sigma_{12}^2 Iq_2, \sigma_3^2 Iq_3, \dots, \sigma_r^2 Iq_{r1}\ )\right)$ ..(8)

$cov(U, e) = 0$

$y = \sim N(F\mu, V)$

$y = (y_1, y_2, \dots, y_N)'$

$var(y) = V = \sum_{i=1}^{r+1} \sigma_e^2 D_i D_i^2 + \sigma_e^2 I_N$ … (9)

On this basis we get the equation:

$= \sum_{i=1}^{r+1} \sigma_i^2 D_i D_i'$

Where : $\sigma_4^2 = \sigma_e^2$ and $D_4 D_4{}' = I_N$

The matrices (D) in equation (7) are known as values [11]and [12] and (6) can be written by matrix form as in:

$$Y = F\mu + D_1U_1 + D_2U_2 + D_3U_3 + e \text{ ..(10)}$$

Where (F) is an integer of units and observations written as integer row by row and matrices D_i are calculated using the multiplication as follows:

$D_1 = I_n \otimes I_b \otimes I_f$

$D_2 = I_n \otimes I_b \otimes I_f$

$D_3 = I_n \otimes I_b \otimes I_f$

$D_4 = I_n \otimes I_b \otimes I_f$

It is possible to get rid of the μ parameter and it is multiplied by (10) in the M projection matrix that has the following formula:

$M = I - F(F'F)^{-1}F'$

$MY = MF\mu + MD_1U_1 + MD_2U_2 + MD_3U_3 + Me$ ..(11)

Since MF=0 is based on the projection matrix property M ,we have:

$MY = MD_1U_1 + MD_2U_2 + MD_3U_3 + Me$

Assume that $MY = X$ therefore,the general linear model is as follows:

$X = MD_1U_1 + MD_2U_2 + MD_3U_3 + Me$ … (12)

$E(X) = 0$, $E(Me) = ME(e) = 0$

$var(X) = MD_1 var(U_1)(MD_1)' + MD_2 var(U_2)(MD_2)' + MD_3 var(U_3)(MD_3)' + M var(e) M'$

Assume that:

$G_i = MD_i$, $i = 1,2.3$

$var(X) = G_1 dig\sigma_1^2 I_{q_1} G_1' + G_2 dig\sigma_2^2 I_{q_2} G_1' + G_3 dig\sigma_3^2 I_{q_3} G_1' + M dig\sigma_4^2 I_N M'$

$M = M' = M^2$

$var(X) = \sigma_1^2 G_1 G_1' + \sigma_2^2 G_2 G_2' + \sigma_3^2 G_3 G_3' + \sigma_1^2 M$

Assume that:

$v_i = G_i G_i'$, $i = 1,2,3$

$var(X) = \sigma_1^2 v_1 + \sigma_2^2 v_2 + \sigma_3^2 v_3 + \sigma_4^2 v_4$

Where $v_4 = M$
Assume that:
$\theta_i = \sigma_i^2 \quad , \quad (i = 1,2,3,4)$
since $var(X)$ is a function by the parameters θ_i and can be expressed as $var(X) = \sum(\theta)$ and the matrix $\sum(\theta)$ can be written as:

$$\sum(\theta) = \theta_1\mu_1 + \theta_2\mu_2 + \theta_3\mu_3 + \theta_4\mu_4 \ ...(13)$$

The matrix $\sum(\theta)$ is a special case of the sample suggested [8] and the parameters are estimated by linear function estimator, [13]

$$a(X) = l_1\theta_1 + l_2\theta_2 + l_3\theta_3 + l_4\theta_4 = l\acute{}\theta \ ...(14)$$

Where $l = (l_1, l_2, l_3, l_4)'$ and $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)'$ and that the function estimator $a(X)$ will be $\hat{a} = X'HX$. where (H) is a symmetry matrix which is to be found from data and $\hat{a}$ meets the non-bias condition and also minimizing Bayes risk function [14].

If the first distribution function has $G(\theta)$ parameter $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)'$ The quadratic loss function takes the following formula:

$$L(\hat{a}, a) = (\hat{a} - a)^2 \ (15)$$

And that the risk function it as it comes:

$$Q(\hat{a}, a) = E[L(\hat{a}, a)] = E[(\hat{a} - a)^2]$$

While taking the beez risk function $b(\hat{a})$ the following form :

$$b(\hat{a}) = E_\theta[Q(\hat{a}, a)] = E_\theta[E(\hat{a} - a)^2]$$

$$b(\hat{a}) = \int Q(\hat{a}, a)\, G(\theta) d\theta$$

$$= \int_{\theta\in\Omega} E(\hat{a} - a)^2\, G(\theta) d\theta \quad ,$$

$$\Omega = \{\theta: \theta_1, \theta_2, \theta_3, \theta_4 > 0\} \ ......(16)$$

3 -Initial distribution of θ_i parameters:

According to the base of Jeffeys [15] the probabilistic density function of variance compounds $\theta_1, \theta_2, \theta_3, \theta_4$ can be specified, and that is done by supposing the systematic distribution, that is:

$$m(\theta) = \frac{1}{\bar{\theta} - \bar{\bar{\theta}}} \bar{\theta} \le \theta \le \bar{\bar{\theta}} \ ....(17)$$

The maximum range of parameters is represented by $\bar{\bar{\theta}}$, and $\bar{\theta}$ represents the minimum range of parameters by supposing that $\theta_1, \theta_2, \theta_3, \theta_4$ are independent, Where:

$$m(\theta_1, \theta_2, \theta_3, \theta_4) = m(\theta_1)m(\theta_2)\, m(\theta_3)m(\theta_4)$$

so $cov(\theta_i, \theta_j) = 0$, then we get the matrix C which will be prior information to the parameters $\theta_1, \theta_2, \theta_3, \theta_4$ Where [16]

$$C = \begin{bmatrix} \frac{\bar{h}_1 - \bar{\bar{h}}_1}{2} \\ \frac{\bar{h}_2 - \bar{\bar{h}}_2}{2} \\ \frac{\bar{h}_3 - \bar{\bar{h}}_3}{2} \\ \frac{\bar{h}_4 - \bar{\bar{h}}_4}{2} \end{bmatrix} \begin{bmatrix} \frac{\bar{h}_1 - \bar{\bar{h}}_1}{2} & \frac{\bar{h}_2 - \bar{\bar{h}}_2}{2} & \frac{\bar{h}_3 - \bar{\bar{h}}_3}{2} & \frac{\bar{h}_4 - \bar{\bar{h}}_4}{2} \end{bmatrix} + \begin{bmatrix} \frac{(\bar{h}_1 - \bar{\bar{h}}_1)^2}{14} & 0 & 0 & 0 \\ 0 & \frac{(\bar{h}_2 - \bar{\bar{h}}_2)^2}{14} & 0 & 0 \\ 0 & 0 & \frac{(\bar{h}_3 - \bar{\bar{h}}_3)^2}{14} & 0 \\ 0 & 0 & 0 & \frac{(\bar{h}_4 - \bar{\bar{h}}_4)^2}{14} \end{bmatrix} ....(18)$$

$$c = ZZ = (z_{ij})(z_{ij}) \quad , i, j = 1,2,3,4 \ ...(19)$$

Model representation

4 -Minimum norm quadraticestimation in variance components model:

The linear function of variance compounds was estimated by the scientist (c.r.rao) as follows:

$$R_1\sigma_1^2 + R_2\sigma_2^2 + R_3\sigma_3^2 + \cdots + R_i\sigma_i^2 = \sum_{i=1}^{r} R_i\sigma_i^2 \ ..(20)$$

By function number (5) of the variable Y of the Model in equation (3) and the way of finding matrix A is done by:

***-Invariance under translation of (b) parameter**

It is not changed when transferring the (b), [17]and it is an alternative of (b), we consider $s=b=b_0$, where b_0 can be considered known constant

$$y = x(s + b_0) + S\tau \ ...(21)$$

By that equation number (3) becomes $Y - Xb_0 = XS + S\tau$ and the estimator $\sum_{i=1}^{r} R_i\sigma_i^2$ becomes as follows :

$$\sum_{i=1}^{r} R_i\sigma_i^2 = (Y - Xb_0)'H(Y - Xb_0)..(22)$$

However, (22) should have the same numerical value regardless of b_0, so we get:

$$\sum_{i=1}^{r} R_i\sigma_i^2 = (Y - Xb_0)'(HY - HXb_0)$$
$$= (Y' - b_0{}'X')(HY - HXb_0)$$
$$= \acute{Y}HY - \acute{Y}HXb_0 - b_0{}'X'HY + b_0{}'X'HXb_0$$
$$= \acute{Y}HY - b_0{}'X'HY - b_0{}'X'HY + b_0{}'X'HXb_0$$
$$\sum_{i=1}^{r} R_i\sigma_i^2 = (\acute{Y}HY - 2b_0{}'X'HY + b_0{}'HXb_0) \ ...(23)$$

In order that (22) has the same numerical value for $\acute{Y}HY$,then it should be realized .

$$HX = 0 \ ...(24)$$

****- Unbiasedness**:

$$Y'HY = (Xb + S\tau)H(Xb + S\tau)$$
$$= (b'X' + \tau'S')(HXb + HS\tau)$$
$$= b'X'HXb + b'X'HS\tau + \tau'S'HXb + \tau'S'HS\tau$$
$$= b'X'HXb + 2b'X'HS\tau + \tau'S'HS\tau$$

and by(24) this we get

$$Y'HY = \tau'S'HS\tau \ ...(25)$$

besides $Y'HY$ can be considered unbiased estimator to $\sum_{i=1}^{r} R_i\sigma_i^2$, if you

$$E(Y'HY) = E(\tau'S'HS\tau) = \sum_{i=1}^{r} R_i\sigma_i^2 \ ..(26)$$

But note that:

$$E(Y'HY) = E(\tau'S'HS\tau)$$
$$= \sum_{i=1}^{r} E(\tau'_i S_i H S_i \tau_i)$$
$$= \sum_{i=1}^{r} tr(S'_i H S'_i E(\tau_i \tau'_i))$$
$$= \sum_{i=1}^{r} tr(H S_i S'_i)\, \sigma_i^2$$
$$= \sum_{i=1}^{r} tr(H v_i)\, \sigma_i^2$$
$$E(Y'HY) = \sum_{i=1}^{r} R_i\sigma_i^2 \ ...(27)$$

and to be unbiased $tr(Hv_i = F_i)$ should be realized, where $v_i = S_i S_i'$.

*****- Minimum norm**

if the variable (τ) is known, then the unbiased normal estimator to $\sum_{i=1}^{r} R_i\sigma_i^2$ will be:

$$\left(\frac{F_1}{m_1}\right)\tau_1'\tau_1 + \left(\frac{F_2}{m_2}\right)\tau_2'\tau_2 + \cdots + \left(\frac{F_r}{m_r}\right)\tau_r'\tau_r = \tau'\Delta\tau \ ..(28)$$

Where:

$$\Delta = diag\left(\left(\frac{F_1}{m_1}\right)Im_1, \left(\frac{F_2}{m_2}\right)Im_2, \ldots, \left(\frac{F_r}{m_r}\right)Im_r\right)$$

And that Im_r matrix unit with m_r
The proposed estimator to be applied in this research is $\tau'S'HS\tau$, and the difference between (13) and ($Y'HY = \tau'S'HS\tau$) is ($\tau'S'HS\tau - \tau'\Delta\tau$), then we get see [18]:
$\tau'(S'HS - \Delta)\tau$ ... (29)
In order to find the estimator ($\acute{Y}HY$), the (S) matrix should be found from equation (29) so that this difference is as minimal as possible in amount. And since (τ) is not known, then (29) can be minimized by minimizing the amount:
($\|S'HS - \Delta\|$) …(30)
Where: $\|.\|$ represents the matrix norm here Euclidean norm which is defind as the following:
$\|T\|^2 = trTT'$
Square shape$Y'HY$ is said to be (MINQUÉ) to $R_i\sigma_i^2$ where the norm ($\|S'HS - \Delta\|$) be small according to the constraint:

$$\left.\begin{array}{l} HX = 0 \\ tr(HV_i) = F_i \quad i = 1,2,3,\dots,r \end{array}\right\} \text{..(31)}$$

$\|S'HS - \Delta\|)^2 = tr(S'HS - \Delta)(S'HS - \Delta)'$ ... (32)
According to constraint $tr(Hv_i = F_i)$ we get:

$$tr(S'HS\Delta) = tr(HS\Delta S') = tr\left(H\sum_{i=1}^{r}\left(\frac{F_i}{m_i}\right)S_iS'_i\right)$$

$$tr(S'HS\Delta) = \sum_{i=1}^{r}\left(\frac{F_i}{m_i}\right)tr(HVi)$$

$$=\sum_{i=1}^{r}\left(\frac{F_i}{m_i}\right)tr(\Delta) \text{ ... (33)}$$

$trtr(S'HS\Delta)(S'HS - \Delta) = (S'HSS'HS) - 2tr(S'HS\Delta) + tr(\Delta\Delta)$
$= tr(S'HSVHS'S) - 2tr(S'HS\Delta) + tr(\Delta\Delta)$
$= tr(HVHS) - 2tr(\Delta\Delta) + tr(\Delta\Delta)$
and after some algebraic operations we get:
$= tr(HVHV) - (\Delta\Delta)$ ... (34)
Since $tr(\Delta\Delta)$ contains (S), then (manqué) is reduced to minimizing $tr(HVHV)$ according to (31) if V nonsingular and the solution of (H) is given by (rao theorem), [7].

5 -Estimating of variance compounds for proposed model by method (MANQUÉ):

The analysis of the bi-division approach in the presence of the interaction represented in the equation (6), it is significant to consider (a_i) is the effect of the first factor (H) which we assume fixed, and (b_i) is the effect of the second factor (M), and of the assumptions that should be applied is ($\sum_{i=1}^{n} a_i = 0$), where (n) represents a number of the levels of factor (H). [19].
Since it is necessary to estimate the variance compounds then to mention:
$M = 1 - F('F)^{-1}F'$
The meet conditions:
$1 - M = M'$
$2 - MM = M^2 = M$
$3 - MF = 0$
From sample $MY = X$
$X = Me$ ... (35)
In sample (35) there is no need to consider anon-exchangeable property under parameter b conversion because the model (7) was hit in the projection matrix M.
And it is only considered a non-bias property

$X'HX = (Me)'H(Me)$
$X'HX = e'MHMe$ ..(36)
In terms of default variables $X'HX(a, b, \gamma, e)$
$E(X'HX) = E(e'MHMe)$
$= \sum_{i=1}^{4} l_i\theta_i$
put
$E(X'HX) = \sum_{i=1}^{4} E(\eta_iU_i'HU_i\eta_i')$
$= \sum_{i=1}^{4} trU_i'HU_iE(\eta_i\eta_i')$
$= \sum_{i=1}^{4} \theta_i tr(U_i'HU_i)$
$= \sum_{i=1}^{4} \theta_i tr(HV_i)$
That is, by the characteristic to be biased
$= \sum_{i=1}^{4} l_i\theta_i$
Which leads to the constraint
$tr(HV_i) = l_i \quad , \quad i = 1,2,3,4$
And if X have anormal distribution $N(\sim 0, V)$
$var(X'HX) = 2tr(HVHV)$
In this equation it is better to suggest that $tr(HVHV)$ is the same resultas the criterion in the Rao theorem of equation (34) any $tr(HVHV)$ miniaturization theorem in the ($HVHV$) to equation (35) and V you know to be $V(\theta_0)$ where θ_0 are the elementary primary values in to θ variation compounds
$H_i(\theta_0) = V_0^{-1}V_iV_0^{-1}$
$H_1(\theta_0) = V_0^{-1}V_1V_0^{-1}$
$H_2(\theta_0) = V_0^{-1}V_2V_0^{-1}$
$H_3(\theta_0) = V_0^{-1}V_3V_0^{-1}$
$H_4(\theta_0) = V_0^{-1}V_4V_0^{-1}$
$V_0 = \theta_{10}V_1 + \theta_{20}V_2 + \theta_{30}V_3 + \theta_{40}V_4$
θ_{i0} represent primary values to θ_i so the estimate (MINQUÉ) of θ to $\hat{\theta}$, which achieves the theorem of rao that:
$X'H_i(\theta_0)X = \left(\left(H_i(\theta_0)V_j\right)\right)\widehat{\theta} \quad , \; i = 1,2,3,4 \; ; j = 1,2,3,4$

$$\begin{pmatrix} tr\boldsymbol{H_1}(\theta_0)V_1 & tr\boldsymbol{H_1}(\theta_0)V_2 & tr\boldsymbol{H_1}(\theta_0)V_3 & tr\boldsymbol{H_1}(\theta_0)V_4 \\ tr\boldsymbol{H_2}(\theta_0)V_1 & tr\boldsymbol{H_2}(\theta_0)V_2 & tr\boldsymbol{H_2}(\theta_0)V_3 & tr\boldsymbol{H_2}(\theta_0)V_4 \\ tr\boldsymbol{H_3}(\theta_0)V_1 & tr\boldsymbol{H_3}(\theta_0)V_2 & tr\boldsymbol{H_3}(\theta_0)V_3 & tr\boldsymbol{H_3}(\theta_0)V_4 \\ tr\boldsymbol{H_4}(\theta_0)V_1 & tr\boldsymbol{H_4}(\theta_0)V_2 & tr\boldsymbol{H_4}(\theta_0)V_3 & tr\boldsymbol{H_4}(\theta_0)V_4 \end{pmatrix}\hat{\theta} = \begin{bmatrix} X' \; \boldsymbol{H_1}(\theta_0) \; X \\ X' \; \boldsymbol{H_2}(\theta_0) \; X \\ X' \; \boldsymbol{H_3}(\theta_0) \; X \\ X' \; \boldsymbol{H_4}(\theta_0) \; X \end{bmatrix}$$

So that estimate of (manqué) to θ is

$$\hat{\theta} = \left(tr\left(H_i(\theta_0)V_j\right)\right)^{-1}\left(\left(X'H_i(\theta_0)\right)X\right)$$

$$C = \begin{bmatrix} X' \; \boldsymbol{H_1}(\theta_0) \; X \\ X' \; \boldsymbol{H_2}(\theta_0) \; X \\ X' \; \boldsymbol{H_3}(\theta_0) \; X \\ X' \; \boldsymbol{H_4}(\theta_0) \; X \end{bmatrix}$$

And put them in vector D ,that is,

$$\boldsymbol{D} = \begin{pmatrix} tr\boldsymbol{H_1}(\theta_0)V_1 & tr\boldsymbol{H_1}(\theta_0)V_2 & tr\boldsymbol{H_1}(\theta_0)V_3 & tr\boldsymbol{H_1}(\theta_0)V_4 \\ tr\boldsymbol{H_2}(\theta_0)V_1 & tr\boldsymbol{H_2}(\theta_0)V_2 & tr\boldsymbol{H_2}(\theta_0)V_3 & tr\boldsymbol{H_2}(\theta_0)V_4 \\ tr\boldsymbol{H_3}(\theta_0)V_1 & tr\boldsymbol{H_3}(\theta_0)V_2 & tr\boldsymbol{H_3}(\theta_0)V_3 & tr\boldsymbol{H_3}(\theta_0)V_4 \\ tr\boldsymbol{H_4}(\theta_0)V_1 & tr\boldsymbol{H_4}(\theta_0)V_2 & tr\boldsymbol{H_4}(\theta_0)V_3 & tr\boldsymbol{H_4}(\theta_0)V_4 \end{pmatrix}$$

$$C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \quad \text{and become our } C = D\hat{\theta}$$

It is important to find the inverse matrix D and LD^{-1}
$\hat{\theta} = D^{-1}C$

$\hat{\theta} = LC$

And also significant to solve equation mode.

and become our $b = LC$

6- Finding standard errore for (manqué) estimator.

$cov(\hat{\theta}) =$
$2\left[Tr\left(H_i(\theta_0)V_j\right)\right]^{-1}\left[TrH_i(\theta_0)V(\hat{\theta})H_j(\theta_0)V(\hat{\theta})\right]\left[Tr\left(H_i(\theta_0)V_j\right)\right]^{-1}$

$$cov(\hat{\theta}) = 2\begin{bmatrix} trH_1(\theta_0)V_1 & trH_1(\theta_0)V_2 & trH_1(\theta_0)V_3 & trH_1(\theta_0)V_4 \\ trH_2(\theta_0)V_1 & trH_2(\theta_0)V_2 & trH_2(\theta_0)V_3 & trH_2(\theta_0)V_4 \\ trH_3(\theta_0)V_1 & trH_3(\theta_0)V_2 & trH_3(\theta_0)V_3 & trH_3(\theta_0)V_4 \\ trH_4(\theta_0)V_1 & trH_4(\theta_0)V_2 & trH_4(\theta_0)V_3 & trH_4(\theta_0)V_4 \end{bmatrix}^{-1}$$

After the variation matrix is found the in puts to the main diameter of this matrix represents variation of variance of effects estimation and by is taking the square root of it and we get the standard errors of vartion variance of effects.

7 -The applied aspect

The applied aspect includes data, random linear model, analyzing data using variance analysis approach from variance analysis table, and estimation by Bayes unbiased estimator and the minimal unbiased quadratic standard estimator depending on the results of variance analysis. The study has been conducted on real data that represent agricultural experiences of two factors, the first is (H) and the second is random (M) with three levels chosen randomly from kinds of tomato and the sample is represented according to equation (5).

Table 1: Tomato Data

factor H / factor M	h_1	h_2	h_3	h_4
m_1	26.22 26.98 17.4	16.22 27.28 17.4	22.22 21.98 17.4	26.22 26.98 17.4
m_2	26.5 11.6 12.3	26.5 21.26 22.13	26.5 12.6 12.3	26.5 11.6 12.13
m_3	19.9 24.8 21.8	19.9 24.8 21.8	29.9 20.8 11.8	19.9 24.8 22.8
m_4	23.7 24.9 19.6	23.7 24.9 19.6	23.7 24.9 19.6	23.7 14.6 19.6
m_5	17.9 28.8 20.6	17.9 28.8 20.6	17.9 28.8 20.6	27.19 28.18 24.16
m_6	12.9 25.1 23.7	12.9 25.1 22.09	22.9 15.11 21.29	22.5 25.71 22.09

It has been assumed that the tomato kind is of big size, and to make it bi-divisional and random in the presence of the interaction, then we must take a random sample from this kind of tomato assuming that the sample size is (28). The random sample has been taken using the number tables and the random sample has been obtained as in table (2).

Table 2: represents the rows and columns of the samples

Factor H / Factor M	$h_1(1)$	$h_2(2)$	$h_4(3)$	Total rows
$m_1(1)$	25.1 23.7 $y_{11\blacksquare}$	17.4 21.26 $y_{12\blacksquare}$	24.16 23.7 $y_{13\blacksquare}$	$y_{1\blacksquare\blacksquare}$
$m_3(2)$	26.22 24.8 $y_{21\blacksquare}$	19.6 22.09 $y_{22\blacksquare}$	25.71 27.19 $y_{23\blacksquare}$	$y_{2\blacksquare\blacksquare}$
$m_5(3)$	20.6 17.9 $y_{31\blacksquare}$	24.9 25.1 $y_{32\blacksquare}$	26.98 14.6 $y_{33\blacksquare}$	$y_{3\blacksquare\blacksquare}$
$m_6(4)$	28.8 19.6 $y_{41\blacksquare}$	12.9 25.1 $y_{42\blacksquare}$	17.4 22.09 $y_{43\blacksquare}$	$y_{4\blacksquare\blacksquare}$
Total of Columns	$y_{\blacksquare 1\blacksquare}$	$y_{\blacksquare 2\blacksquare}$	$y_{\blacksquare 3\blacksquare}$	$y_{\blacksquare\blacksquare\blacksquare}$

Where,

$y_{\blacksquare\blacksquare\blacksquare}$: general total results of all views from the random sample and expressed in symbol $\sum_{i=1}^{h}\sum_{j=1}^{m}\sum_{k=1}^{n} y_{ijk}$

$(\bar{y}_{\blacksquare\blacksquare\blacksquare})$ represents the general medium and is expressed by the symbol $({}^{y_{\blacksquare\blacksquare\blacksquare}}/_{hmn})$, besides

$h = 4$, $m = 3$, $n = 2$

$(y_{i\blacksquare\blacksquare} = \sum_{j=1}^{3}\sum_{k=1}^{2} y_{ijk})$ and

$(y_{\blacksquare j\blacksquare} = \sum_{i=1}^{4}\sum_{k=1}^{2} y_{ijk})$ as well as $(y_{ij\blacksquare} = \sum_{k=1}^{2} y_{ijk})$.

Where,

$y_{i\blacksquare\blacksquare}$: total results of all views of the i.

$y_{\blacksquare j\blacksquare}$: total results of all views of the j.

$y_{ij\blacksquare}$: total views in the cell in row I and column j.

Then the results appeared in the variance analysis table after applying the variance analysis approach (ANOVA) as illustrated in table (3).

Table 3: represent the variance analysis

Variance source $(M.S.)$	Expectation of Mean Square Error $E(M.S.)$	Total of Squares $(S.S.)$	Mean squares $(M.S.)$	Degree of Freedom $(D.F.)$
Among columns	$\hat{\sigma}_1^2$=3.431	210.5549	2.433	5
Among rows	$\hat{\sigma}_2^2$=43.764	10.539	1.085	3
Among columns and rows	$\hat{\sigma}_3^2$=30.543	168.8425	8.665	6
Error	$\hat{\sigma}_4^2$=0.6230	4.111	1.841	11
Total		261.004		25

Then the first estimation of the parameters has been applied which is the unbiased quadratic Bayes estimator. [The estimations we have obtained via variance analysis are elementary values of the parameters and the first distribution of the parameters is considered systematic and due to that have chosen to consider that distribution elementary and through it have known the first distributions of parameters and thus we find the first and second factors]. We have gotten the first estimations by using (ANOVA), and we have come up with the estimations by using Bayes approach and were near to those come up with via analyzing the (ANOVA) with the hypothesis ($\theta_i = \sigma_i^2$), table (4) illustrates the results of these estimations and also the elementary estimations by using variance analysis approach:

Table 4: illustrates the elementary estimations via variance analysis

$\hat{\sigma}_1^2$	$\hat{\sigma}_2^2$	$\hat{\sigma}_3^2$	$\hat{\sigma}_4^2$
3.431	43.764	30.543	0.6230
$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$
3.431	43.764	30.543	0.6230

As for the results of unbiased Bayes estimations of the random linear sample, they are as follows: table number (5). Also, initial information of parameters $\theta_1, \theta_2, \theta_3, \theta_4$, are systematically distributed and by using the equations (17) we get:

$m(\theta_1) = f(\theta_1) = {}^{1}/_{6.7751} = 0.1475$

$m(\theta_2) = f(\theta_2) = {}^{1}/_{11.4123} = 0.0877$

$m(\theta_4) = f(\theta_4) = {}^{1}/_{1.4412} = 0.6938$

By using (18) we get the matrix:

$$c = \begin{bmatrix} 0.5419 & 0 & 4.667 & 0.4124 \\ 0 & 0 & 0 & 0.4195 \\ 8.1262 & 0 & 41.744 & 1.7730 \\ 0.3964 & 1.5570 & 0 & 0.3339 \end{bmatrix}$$

Table 5: unbiased Basyan estimation results of the random linear model

$\hat{\sigma}_1^2$	$\hat{\sigma}_2^2$	$\hat{\sigma}_3^2$	$\hat{\sigma}_4^2$
2.106	1.211	7.605	1.111

After that, the second sample of the estimation is applied which is the minimal unbiased quadratic standard of the bi-division variance analysis sample. have used the same real data to apply the second sample in the interaction to get the estimated values of the variance compounds these estimates of variation compounds have been obtained and will represent the primary values of the variation compounds as follows:

$\hat{\sigma}_1^2 = \hat{\theta}_1 = 3.431$

$\hat{\sigma}_2^2 = \hat{\theta}_2 = 43.764$

$\hat{\sigma}_3^2 = \hat{\theta}_3 = 30.543$

$\hat{\sigma}_4^2 = \hat{\theta}_4 = 0.6230$

$H_{01}: M_1 = M_2 = M_3 = M_4$

H_{11} : at least two of them are unequal

$H_{11}: \hat{\sigma}_1^2 = 0 \quad , \quad \hat{\sigma}_2^2 = 0 \quad , \quad \hat{\sigma}_3^2 = 0 \quad , \quad \hat{\sigma}_4^2 = 0$

$H_{12}: \hat{\sigma}_1^2 \neq 0 \quad , \quad \hat{\sigma}_2^2 \neq 0 \quad , \quad \hat{\sigma}_3^2 \neq 0 \quad , \quad \hat{\sigma}_4^2 \neq 0$

By arribut values at amorale level 0.01there no significant differences in fertilizer types and there are very significant differences for tomato varieties there are very significant differences in the interaction between fertilizer type and tomato class .

The repetitive style is used to estimate the minimal unbiased quadratic standard. The estimation results in each cycle are considered elementary values of the subsequent repetition until we get divergence and we have gotten the latter and the difference between the last two repetitions is 10 -10. Also, the standard errors are calculated and results are shown in table (3), as well as the initial estimation of variance compounds found in table number (4) by using the minimal unbiased standard of the random linear sample, by this style have gotten estimations as shown in table (6).

Table 6: Estimation of variance compounds by minimal quadratic unbiased bi-division standard in comparison with parameters estimation using baque esimation method

Variance compounds	Standard Error	baque esimation	Manque estimation	Estimation of variance analysis
$\hat{\theta}_1$	13.6664309	2.106	4.6706	3.431
$\hat{\theta}_2$	6.77430977	1.211	11.221	43.764
$\hat{\theta}_3$	10.1198230	7.605	3.647	30.543
$\hat{\theta}_4$	17.6104320	1.111	5.101	0.6230

Table (6) shows an estimate of (MANQUÉ) for $\theta_1, \theta_2, \theta_3, \theta_4$ parameters also with the standard error calculation parameters $\theta_1, \theta_2, \theta_3, \theta_4$.

Not that there is difference of between estimate (MANQUÉ) and (baque) but they both lead to the rejection of the hypothesis.

When compared with the results by (baque) as in table (5), it is noticed that there are clear differences between the two estimations when compared with those of the unbiased Bays method where the results were more encouraging.

8 -Conclusion

In this study, information estimation of bi-division linear model with the existence of interaction has been conducted. The estimation has been done by (MANQUÉ) and (BAQUE) consists of four parameters represented by (σ_1^2) which is factor effect variance compound (H), and (σ_4^2) is factor effect variance on compound (M), and (σ_3^2) is interaction effect variance compound, and (σ_4^2) is random error effective compound. The study is conducted on real data representing developing the growing of tomato which holds three factors: the first factor is the kind of tomato, the second one is the kind of fertilizer used, while the third one is the interaction. The data have been represented in table (1), the sample size is supposed to be (28), the random sample has been taken via numbers table as is shown in table (2). Then, the variance analysis approach (A NOVA) has been applied, the results are shown in table (3) and the estimations have been considered initial values for the parameters as shown in table (4); after that the unbiased estimation of Bayes (BAQUE), which has already been illustrated in the theoretical part; has been applied and the estimation results are illustrated via Bayes style compared with variance analysis as in table (5) which almost compatible. Then, the parameters have been estimated by minimal unbiased quadratic standard together with standard error of parameters (θ_1 ,θ_2 , $\theta_{3,}$ θ_4) as is shown in table (6) where there is a difference between (MANQUÉ) and (BAQUE) estimation. Out of these results, have come up with the fact that the estimations conducted by the unbiased Bayes estimator style are much better than finding these estimations via minimal unbiased quadratic standard.

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مقارنة بين تقدير بيز وتقدير اصغر معيار تربيعي غير متحيز لنموذج تحليل التباين ثنائي التقسيم بوجود التفاعل

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الملخص

تم في هذه الدراسة تقدير معلمات مركبات التباين لنموذج تحليل التباين المختلط ثنائي التقسيم، تم الحصول على هذا التقدير بواسطة تقدير معلمات مركبات التباين بواسطة مقدر بيز غير المتحيز التربيعي bayes quadratic unbiased estimator (baque). والطريقة الثانية لتقدير معلمات مركبات التباين عن طريق تقدير اصغر معيار تربيعي غير متحيز (minimum norm quadratic unbiased estimation) (MINQUÉ) ، وقد تم توثيق طريقة التقدير على بيانات حقيقية تم الحصول عليها من اقسام كلية الزراعة/ جامعة الموصل، تمثل هذه البيانات تطوير زراعة اصناف الطماطة بحيث يمثل التطوير ثلاثة عوامل هي عامل صنف الطماطة و هذا يمثل العامل الاول H وعامل نسبة السماد الطبيعي ويمثل العامل الثاني M والتفاعل موجود بين العاملين HM ،تم سحب عينة عشوائية من هذه البيانات من اجل الحصول على النموذج الخطي العشوائي، حيث ان القيم الابتدائية التي تم تقديرها بواسطة مقدر بيز الغير متحيز مقاربة جدا للتقديرات التي تم تقديرها باسلوب تحليل التباين مقارنة بالقيم المقدرة لمعلمات تقدير التباين التي تم تقديرها بواسطة تقدير اصغر معيار تربيعي غير المتحيز ،حيث ان القيم الابتدائية والتي تمثل معلمات النموذج الخطي العشوائي المستخدمة في تقدير اصغر معيار تربيعي غير المتحيز (manqué) حيث تم الحصول على القيم الابتدائية للتقديرات ايضا عن طريق تحليل التباين ثنائي التقسيم بعدها نستخدم هذه التقديرات كقيم ابتدائية في تقدير اصغر معيار تربيعي غير متحيز حيث تم الحصول على التقارب في التكرار الخامس، تمت مقارنة النتائج النهائية بين طريقة التقدير بواسطة اسلوب بيز وبين التقديرات بواسطة اصغر معيارتربيعي غير متحيز حيث كانت نتائج التقديرات بواسطة اسلوب بيزقريبة جدا من التقديرات بواسطة تحليل التباين.