IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces

Fatimah M. Mohammed , Shaymaa F. Motar

Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq

https://doi.org/10.25130/tjps.v24i3.382

ARTICLE INFO.

Article history:

-Received: 30 / 4 / 2018

-Accepted: 28 / 5 / 2018

-Available online: / / 2019

Keywords: fuzzy neutrosophic set, fuzzy neutrosophic-$\tau_{0,1}$continuous, fuzzy neutrosophic $\tau_{0,2}$continuous, fuzzy neutrosophic-$\tau_{0,1}$contra continuous, fuzzy neutrosophic-$\tau_{0,2}$contra continuous functions.

Corresponding Author:

Name: **Fatimah M. Mohammed**

E-mail: nafea_y2011@yahoo.com

Tel:

ABSTRACT

In this paper, we defined fuzzy neutrosophic-$\tau_{0,1}$continuous, fuzzy neutrosophic-$\tau_{0,2}$continuous, fuzzy neutrosophic-$\tau_{0,1}$contra continuous and fuzzy neutrosophic-$\tau_{0,2}$contra continuous functions. Then, we define the relationship between the define functions and studied functions with their comparative.

1- Introduction

The concept of fuzzy sets (FS, for short) was introduced by Zadeh in 1965 [1]. Then the fuzzy set theory are extension by many researchers. Intutionistic fuzzy sets (IFS, for short) was one of the extension sets by K. Atanassov in 1983 [2,3,4], when fuzzy set give the degree of membership function of an element in the sets. Then, the intuitionistic fuzzy sets give a degree of membership function and a degree of non-membership function. After that, several researches were conducted on the generalizations of the notion of intuitionistic fuzzy sets. The concept of neutrosophy, neutrosophic set and neutrosophic component was F. Smarandache in 1999 [5]. Then the concept of neutrosophic set (NS, for short) and neutrosophic topological space (NTS, for short) define by A. A. Salama and S.A. Alblowi 2012 [7]. In the year 2013 by I. Arockiarani, I. R.Sumathi and J. Martina Jency [8] define the fuzzy neutrosophic set. Next, in the year 2014 by I. Arockiarani and J. Martina Jency [9] define the fuzzy neutrosophic topological space.

The fuzzy neutrosophic sets was define with membership, non-membership and indeterminacy degrees. In the last year, (2017) by Y. Veereswari [10] introduced of fuzzy neutrosophic continuous function.

2. Some Basic of Topological Concepts

Definition 2.1 [8, 10]: Let X be a non-empty fixed set. Fuzzy neutrosophic set (FNS, for short), λ_N is an object having the form $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ where the functions $\mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N} : X \to [0, 1]$ denote the degree of membership function (namely $\mu_{\lambda N}(x)$), the degree of indeterminacy function (namely $\sigma_{\lambda N}(x)$) and the degree of non-membership (namely $\nu_{\lambda N}(x)$) respectively, of each set λ_N we have, $0 \le \mu_{\lambda N}(x) + \sigma_{\lambda}(x) + \nu_{\lambda N}(x) \le 3$, for each $x \in X$.

Remark 2.2 [10]: FNS $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ can be identified to an ordered triple $<x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ in [0, 1] on X.

Definition 2.3 [10]: Let X be a non-empty set and the FNSs λ_N and β_N be in the form $\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$ and, $\beta_N = \{< x, \mu_{\beta N}(x), \sigma_{\beta N}(x), \nu_{\beta N}(x) >: x \in X\}$ on X. Then**:**

i. $\lambda_N \subseteq \beta_N$ iff $\mu_{\lambda N}(x) \le \mu_{\beta N}(x)$, $\sigma_{\lambda N}(x) \le \sigma_{\beta N}(x)$ and $\nu_{\lambda N}(x) \ge \nu_{\beta N}(x)$ for all $x \in X$,

ii. $\lambda_N = \beta_N$ iff $\lambda_N \subseteq \beta_N$ and $\beta_N \subseteq \lambda_N$,

iii. $1_N - \lambda_N = \{< x, \nu_{\lambda N}(x), 1 - \sigma_{\lambda N}(x), \mu_{\lambda N}(x) >: x \in X\}$,

iv. $\lambda_N \cup \beta_N = \{< x, Max(\mu_{\lambda N}(x), \mu_{\beta N}(x)), Max(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), Min(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$, **v.** $\lambda_N \cap \beta_N = \{< x, Min(\mu_{\lambda N}(x), \mu_{\beta N}(x)), Min(\sigma_{\lambda N}(x), \sigma_{\beta N}(x)), Max(\nu_{\lambda N}(x), \nu_{\beta N}(x)) >: x \in X\}$,
vi. $[\,]\lambda_N = \{< x, \mu_{\lambda N}(x), \sigma_{\lambda N}(x), 1-\mu_{\lambda N}(x) >: x \in X\}$,
vii. $<>\lambda_N = \{< x, 1-\nu_{\lambda N}(x), \sigma_{\lambda N}(x), \nu_{\lambda N}(x) >: x \in X\}$,
viii. $0_N = < x, 0, 0, 1>$ and $1_N = <x, 1, 1, 0>$.

Definition 2.4 [10]: Fuzzy neutrosophic topology (FNT, for short) on a non-empty set X is a family τ of fuzzy neutrosophic subsets in X satisfying the following axioms.
i. $0_N, 1_N \in \tau$,
ii. $\lambda_{N1} \cap \lambda_{N2} \in \tau$ for any $\lambda_{N1}, \lambda_{N2} \in \tau$,
iii. $\cup \lambda_{N\,j} \in \tau, \forall \{\lambda_{N\,j} : j \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space (FNTS, for short). The elements of τ are called fuzzy neutrosophic open sets (FN-open set, for short). The complement of FN-open set in the FNTS (X, τ) is called fuzzy neutrosophic closed set (FN-closed set, for short).

Definition 2.5 [9]: Let (X, τ) be FNTS and $\lambda_N = <x, \mu_{\lambda N}, \sigma_{\lambda N}, \nu_{\lambda N}>$ is FNS in X. Then, the fuzzy neutrosophic-closure (FNCl, for short) and fuzzy neutrosophic-Interior of λ_N (FNInt, for short) are defined by:
$FNCl(\lambda_N) = \cap\{\beta_N : \beta_N$ is FN-closed set in X and $\lambda_N \subseteq \beta_N\}$, $FNInt(\lambda_N) = \cup\{\beta_N : \beta_N$ is FN-open set in X and $\beta_N \subseteq \lambda_N\}$.
Note that $FNCl(\lambda_N)$ is FN-closed set and $FNInt(\lambda_N)$ is FN-open set in X. Further,
i. λ_N is FN-closed set in X iff $FNCl(\lambda_N) = \lambda_N$,
ii. λ_N is FN-open set in X iff $FNInt(\lambda_N) = \lambda_N$.

Definition 2.6 [10]: Let (X, τ) be FNTS on X. Then
i. $FN\tau_{0,1} = \{[\,]\lambda_N : \lambda_N \in \tau\}$,
ii. $FN\tau_{0,2} = \{<>\lambda_N : \lambda_N \in \tau\}$are FNT on X.

Definition 2.7 [10]: If $\beta_N = \{<y, \mu_{\beta N}(y), \sigma_{\beta N}(y), \nu_{\beta N}(y)>: y \in Y\}$ is FNS in Y. Then, the inverses image of β_N under f, ($f^{-1}(\beta_N)$, for short) is FNS in X defined by $f^{-1}(\beta_N) = \{< x, f^{-1}(\mu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(\nu_{\beta N})(x) >: x \in X\}$ where, $f^{-1}(\mu_{\beta N})(x) = \mu_{\beta N} f(x)$, $f^{-1}(\sigma_{\beta N})(x) = \sigma_{\beta N} f(x)$ and $f^{-1}(\nu_{\beta N})(x) = \nu_N f(x)$.

Definition 2.8 [10]: Let (X, τ_x) and (Y, τ_y) are two FNTSs. Then a function $f : (X, \tau_x) \to (Y, \tau_y)$ is called fuzzy neutrosophic-continuous (FN-con., for short) if the inverses image of every FN-open (FN-closed) set in (Y, τ_y) is FN-open (FN-closed) set in (X, τ_x).

Definition 2.9 [6]: Let (X, τ_x) and (Y, τ_y) are two FNTSs. Then a function $f : (X, \tau_x) \to (Y, \tau_y)$ is called fuzzy neutrosophic-contra continuous (FN-ccon., for short) if the inverses image of every FN-open (FN-closed) set in (Y, τ_y) is FN-closed (FN-open) set in (X, τ_x).

Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces

Now, we introduced a new concept in fuzzy netrosophoic topological spaces and called it fuzzy neutrosophic-$\tau_{0,1}$continuous, fuzzy neutrosophic-$\tau_{0,2}$ continuous, fuzzy neutrosophic-$\tau_{0,1}$contra continuous and fuzzy neutrosophic-$\tau_{0,2}$contra continuous functions.

Definition 3.1: Let $(X, FN\tau_{x0,1})$ and $(Y, FN\tau_{y0,1})$ are two FNTSs. Then:
i. A function $f : (X, FN\tau_{x0,1}) \to (Y, FN\tau_{y0,1})$ is called fuzzy neutrosophic- $\tau_{0,1}$continuous (FN-$\tau_{0,1}$con., for short) if the inverse image of every FN-open (FN-closed) set in $(Y, FN\tau_{y0,1})$ is FN-open (FN-closed) set in $(X, FN\tau_{x0,1})$.
ii. A function $f: (X, FN\tau_{x0,2}) \to (Y, FN\tau_{y0,2})$ is called fuzzy neutrosophic- $\tau_{0,2}$continuous (FN-$\tau_{0,2}$con., for short) if the inverse image of every FN-open (FN-closed) set in $(Y, FN\tau_{y0,2})$ is FN-open (FN-closed) set in $(X, FN\tau_{x0,2})$.

Example 3.2: 1- Let X = Y = {a, b}define FNSs λ_N in X and β_N in Y as follows:
$\lambda_N = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) >$. The family, $\tau_x = \{0_N, 1_N, \lambda_N\}$ is FNT.
And $\beta_N = <y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$. The family, $\tau_y = \{0_N, 1_N, \beta_N\}$ is FNT.
Define $f : (X, \tau_x) \to (Y, \tau_y)$ as follows: f(a) = b and f(b) = a.
If, $\beta_N = < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$ is FN-open set in τ_y.
Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) > \in \tau_x$.
So, $f^{-1}(\beta_N)$ is FN-open set in τ_x. Hence, f is (FN-con.) function.

2- Take, **(1)** so from τ_x we get:
The family, $FN\tau_{x0,1} = \{0_N, 1_N, < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) >\}$ is FNT.
And, from τ_y we get:
The family, $FN\tau_{y0,1} = \{0_N, 1_N, < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >\}$ is FNT.
Define $f : (X, FN\tau_{x0,1}) \to (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.
Now, let $\eta_N = < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >$ is FN-open set in $FN\tau_{y0,1}$.
Then, $f^{-1}(\eta_N) = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) > \in FN\tau_{x0,1}$.
So, $f^{-1}(\eta_N)$ is FN-open set in $FN\tau_{x0,1}$. Hence, f is (FN-$\tau_{0,1}$con.) function.

3- Take, **(1)** so from τ_x we get:
The family, $FN\tau_{x0,2} = \{0_N, 1_N, < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) >\}$ is FNT.
And, from τ_y we get:
The family, $FN\tau_{y0,2} = \{0_N, 1_N, < y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >\}$ is FNT.
Define $f : (X, FN\tau_{x0,2}) \to (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.
If, $\Psi_N = <y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$ is FN-open set in $FN\tau_{y0,2}$.

Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6})> \in FN\tau_{x0,2}$.
So, $f^{-1}(\Psi_N)$ is FN-open set in $FN\tau_{x0,2}$. Hence, f is (FN-$\tau_{0,2}$con.) function.

Theorem 3.3:
Let (X, τ_x), (Y, τ_y) two FNTSs and f: $(X, \tau_x) \to (Y, \tau_y)$ is a function.
i. If, f is (FN-con.) function. Then, f is (FN-$\tau_{0,1}$con.) function.
ii. If, f is (FN-con.) function. Then, f is (FN-$\tau_{0,2}$con.) function.

Proof:
i. Let f be (FN-con.) function. Then,
$\beta_N = \{< y, \mu_{\beta N}(y), \sigma_{\beta N}(y), \nu_{\beta N}(y) >: y\in Y \}$ is FN-open set in τ_y, so
$f^{-1}(\beta_N) = \{< x, f^{-1}(\mu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(\nu_{\beta N})(x) >: x \in X\}$, where
$f^{-1}(\mu_{\beta N})(x) = \mu_{\beta N} f(x)$, $f^{-1}(\sigma_{\beta N})(x) = \sigma_{\beta N} f(x)$ and $f^{-1}(\nu_{\beta N})(x) = \nu_{\beta N} f(x)$
is FN-open set in τ_x. And, by **Definition 2.8** we get:
$\eta_N = \{ < y, \mu_{\beta N}(y), \sigma_{\beta N}(y), 1-\mu_{\beta N}(y) > : y\in Y \}$ is FN-open set in
$FN\tau_{y0,1}$, so $f^{-1}(\eta_N) =\{<x, f^{-1}(\mu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(1- \mu_{\beta N})(x) >: x\in X\}$
$= \{<x, f^{-1}(\mu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), 1- f^{-1}(\mu_{\beta N})(x) >: x \in X\}$ is FN-open
set in $FN\tau_{x0,1}$. By **Definition 3.1 (i)**. Hence, f is (FN-$\tau_{0,1}$con.) function.
ii. Let f be (FN-con.) function. Then,
$\beta_N = \{< y, \mu_{\beta N}(y), \sigma_{\beta N}(y), \nu_{\beta N}(y) > : y\in Y \}$ is FN-open set in τ_y, so
$f^{-1}(\beta_N) = \{< x, f^{-1}(\mu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(\nu_{\beta N})(x) > : x \in X\}$, where
$f^{-1}(\mu_{\beta N})(x) = \mu_{\beta N} f(x)$, $f^{-1}(\sigma_{\beta N})(x) = \sigma_{\beta N} f(x)$ and $f^{-1}(\nu_{\beta N})(x) = \nu_N f(x)$
is FN-open set in τ_x. And, by **Definition 2.8** we get:
$\Psi_N =\{< y, 1-\nu_{\beta N}(y), \sigma_{\beta N}(y), \nu_{\beta N}(y) > : y\in Y \}$ is FN-open set in $FN\tau_{y0,2}$,
so $f^{-1}(\Psi_N) = \{ < x, f^{-1}(1-\nu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(\nu_{\beta N})(x) > : x \in X\}$
$= \{<x, 1- f^{-1}(\nu_{\beta N})(x), f^{-1}(\sigma_{\beta N})(x), f^{-1}(\nu_{\beta N}) >: x \in X\}$ is FN-open set in
$FN\tau_{x0,2}$. By **Definition 3.1 (ii)**. Hence, f is (FN-$\tau_{0,2}$con.) function.

Remark 3.4:
The convers of **Theorem 3.3** is not true in general and we can show it by the following example.

Example 3.5: i. Let X =Y = {a, b } define FNSs λ_N in X and β_N in Y as follows:
$\lambda_N = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}) >$.The family, $\tau_x = \{0_N, 1_N, \lambda_N \}$ is FNT.
And, $\beta_N = <y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) >$. The family, $\tau_y =\{0_N, 1_N, \beta_N\}$ is FNT.
Define f : $(X, \tau_x) \to (Y, \tau_y)$ as follows: f(a) = b and f(b) = a.
If, $\beta_N = < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.7}) >$ is FN-open set in τ_y.
Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.7}, \frac{b}{0.4}) > \notin \tau_x$.
Hence, f is not (FN-con.) function.
But, from τ_x we get:
The family, $FN\tau_{x0,1} = \{0_N, 1_N, < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) >\}$is FNT.
And, from τ_y we get:
The family, $FN\tau_{y0,1} =\{0_N, 1_N, < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >\}$ is FNT.
Define f: $(X, FN\tau_{x0,1}) \to (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.
If, $\eta_N = < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >$ is FN-open set in $FN\tau_{y0,1}$.
Then, $f^{-1}(\eta_N) = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) > \in FN\tau_{x0,1}$.
So, $f^{-1}(\eta_N)$ is FN-open set in $FN\tau_{x0,1}$. Hence, f is (FN-$\tau_{0,1}$con.) function.
ii. Let X = Y = a, b } define FNSs λ_N in X and β_N in Y as follows :
$\lambda_N = <x, (\frac{a}{0.1}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) >$. The family, $\tau_x =\{0_N, 1_N, \lambda_N \}$ is FNT.
$\beta_N = <y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$. The family, $\tau_y =\{0_N, 1_N, \beta_N \}$ is FNT.
Define f : $(X, \tau_x) \to (Y, \tau_y)$ as follows: f(a) = b and f(b) = a.
If, $\beta_N= < y, (\frac{a}{0.2}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$ is FN-open set in τ_y.
Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.6}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) > \notin \tau_x$.
Hence, f is not (FN-con.) function.
But, from τ_x we get:
The family, $FN\tau_{x0,2} =\{0_N, 1_N, < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) >\}$ is FNT.
And, from τ_y we get:
The family, $FN\tau_{y0,2} =\{0_N, 1_N, < y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >\}$ is FNT.
Define f : $(X, FN\tau_{x0,2}) \to (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.
If, $\Psi_N = < y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$ is FN-open set in $FN\tau_{y0,2}$.
Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) > \in FN\tau_{x0,2}$.
So, $f^{-1}(\Psi_N)$ is FN-open set in $FN\tau_{x0,2}$. Hence, f is (FN-$\tau_{0,2}$con.) function.

Remark 3.6:
The relation between (FN-$\tau_{0,1}$con.) and (FN-$\tau_{0,2}$con.) functions are independent and we can show it by the following example.

Example 3.7:
1- Take, **Example 3.5 (i)**. Then, f is (FN-$\tau_{0,1}$con.) function.

But, f is not (FN-$\tau_{0,2}$con.) function. Since, from τ_x we get:

The family, $FN\tau_{x0,2}=\{0_N, 1_N, < x, (\frac{a}{0.7},\frac{b}{0.4}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.3},\frac{b}{0.6}) >\}$ is FNT.

And, from τ_y we get:

The family, $FN\tau_{y0,2}=\{0_N, 1_N, < y, (\frac{a}{0.6},\frac{b}{0.3}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.7}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.

If, $\Psi_N = < y, (\frac{a}{0.6},\frac{b}{0.3}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.7}) >$ is FN-open set in $FN\tau_{y0,2}$.

Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.3},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.7},\frac{b}{0.4}) > \notin FN\tau_{x0,2}$.

2- Take, **Example 3.5 (ii)**. Then, f is (FN-$\tau_{0,2}$con.) function.

But, f is not (FN-$\tau_{0,1}$con.) function. Since, from τ_x we get:

The family, $FN\tau_{x0,1}=\{0_N, 1_N, <x, (\frac{a}{0.1},\frac{b}{0.5}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.9},\frac{b}{0.5}) >\}$ is FNT.

And, from τ_y we get:

The family, $FN\tau_{y0,1}=\{0_N, 1_N, <y, (\frac{a}{0.2},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.8},\frac{b}{0.4}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.

If, $\eta_N = < y, (\frac{a}{0.2},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.8},\frac{b}{0.4}) >$ is FN-open set in $FN\tau_{y0,1}$.

Then, $f^{-1}(\eta_N) = < x, (\frac{a}{0.6},\frac{b}{0.2}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.8}) > \notin FN\tau_{x0,1}$.

Definition 3.8:

Let $(X, FN\tau_{x0,1})$ and $(Y, FN\tau_{y0,1})$ are two FNTSs. Then:

i. A function f: $(X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$ is called fuzzy neutrosophic- $\tau_{0,1}$contra continuous (FN-$\tau_{0,1}$ccon., for short) if the inverse image of every FN-open (FN-closed) set in $(Y, FN\tau_{y0,1})$ is FN- closed (FN-open) set in $(X, FN\tau_{x0,1})$.

ii. A function f : $(X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$ is called fuzzy neutrosophic-$\tau_{0,2}$contra continuous (FN-$\tau_{0,2}$ccon., for short) if the inverse image of every FN-open (FN-closed) set in $(Y, FN\tau_{y0,2})$ is FN-closed (FN-open) set in $(X, FN\tau_{x0,2})$.

Example 3.9: 1- Let X=Y= {a, b }define FNSs λ_N in X and β_N in Y as follows:

$\lambda_N = <x, (\frac{a}{0.9},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.5})>$. The family, $\tau_x = \{0_N, 1_N, \lambda_N\}$ is FNT.

Such that, $1_N\text{-}\tau_x =\{1_N, 0_N, < x, (\frac{a}{0.4},\frac{b}{0.5}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.9},\frac{b}{0.6}) >\}$.

And, $\beta_N = < y, (\frac{a}{0.5},\frac{b}{0.4}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.6},\frac{b}{0.9}) >$.The family, $\tau_y =\{0_N, 1_N, \beta_N\}$ is FNT.

Define f : $(X, \tau_x) \rightarrow (Y, \tau_y)$ as follows: f(a) = b and f(b) = a.

If, $\beta_N = <y, (\frac{a}{0.5},\frac{b}{0.4}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.6},\frac{b}{0.9}) >$ is FN-open set in τ_y.

Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.4},\frac{b}{0.5}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.9},\frac{b}{0.6}) > \in 1_N\text{-}\tau_x$.

So, $f^{-1}(\beta_N)$ is FN-closed set in τ_x. Hence, f is (FN-ccon.) function.

2- Let X = Y = {a, b } define FNSs λ_N in X and β_N in Y as follows:

$\lambda_N = < x, (\frac{a}{0.4},\frac{b}{0.2}), (\frac{a}{0.6},\frac{b}{0.5}), (\frac{a}{0.5},\frac{b}{0.7}) >$.The family, $\tau_x =\{0_N, 1_N, \lambda_N\}$ is FNT.

From τ_x we get:

The family, $FN\tau_{x0,1} = \{ 0_N, 1_N, <x, (\frac{a}{0.4},\frac{b}{0.2}), (\frac{a}{0.6},\frac{b}{0.5}), (\frac{a}{0.6},\frac{b}{0.8}) > \}$ is FNT.

Such that, $1_N\text{-}FN\tau_{x0,1}= \{1_N, 0_N, < x, (\frac{a}{0.6},\frac{b}{0.8}), (\frac{a}{0.4},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.2}) >\}$.

And, $\beta_N = < y, (\frac{a}{0.8},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.4}), (\frac{a}{0.4},\frac{b}{0.3}) >$.The family, $\tau_y =\{0_N, 1_N, \beta_N \}$ is FNT.

From τ_y we get:

The family, $FN\tau_{y0,1} =\{0_N, 1_N, <y, (\frac{a}{0.8},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.4}), (\frac{a}{0.2},\frac{b}{0.4}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,1}) \rightarrow (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.

If, $\eta_N = < y, (\frac{a}{0.8},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.4}), (\frac{a}{0.2},\frac{b}{0.4}) >$ is FN-open set in $FN\tau_{y0,1}$.

Then, $f^{-1}(\eta_N) = <x, (\frac{a}{0.6},\frac{b}{0.8}), (\frac{a}{0.4},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.2})> \in 1_N\text{-} FN\tau_{x0,1}$.

So, $f^{-1}(\eta_N)$ is FN-closed set in $FN\tau_{x0,1}$. Hence, f is (FN-$\tau_{0,1}$ccon.) function.

3- Let X = Y = {a, b} define FNSs λ_N in X and β_N in Y as follows :

$\lambda_N = < x, (\frac{a}{0.4},\frac{b}{0.3}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.8},\frac{b}{0.6}) >$.The family, $\tau_x =\{0_N, 1_N, \lambda_N\}$ is FNT.

From τ_x we get:

The family $FN\tau_{x0,2} = \{0_N, 1_N, < x, (\frac{a}{0.2},\frac{b}{0.4}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.8},\frac{b}{0.6}) >\}$ is FNT.

Such that, $1_N\text{-}FN\tau_{x0,2} = \{1_N, 0_N, <x, (\frac{a}{0.8},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.2},\frac{b}{0.4}) >\}$. And,

$\beta_N = <y, (\frac{a}{0.4},\frac{b}{0.7}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.2}) >$. The family, $\tau_y =\{0_N, 1_N, \beta_N\}$ is FNT.

From τ_y we get:

The family, $FN\tau_{y0,2} = \{0_N, 1_N, < y, (\frac{a}{0.6},\frac{b}{0.8}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.2}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,2}) \rightarrow (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.

If, $\Psi_N = <y, (\frac{a}{0.6},\frac{b}{0.8}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.4},\frac{b}{0.2}) >$ is FN-open set in $FN\tau_{y0,2}$.

Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.8},\frac{b}{0.6}), (\frac{a}{0.5},\frac{b}{0.5}), (\frac{a}{0.2},\frac{b}{0.4}) > \in 1_N\text{-} FN\tau_{x0,2}$.

So, $f^{-1}(\Psi_N)$ is FN-closed set in $FN\tau_{x0,2}$. Hence, f is (FN-$\tau_{0,2}$ccon.) function.

Remark 3.10: i. The relation between (FN-ccon.) and (FN-$\tau_{0,1}$ccon.) functions are independent.

ii. The relation between (FN-ccon.) and (FN-$\tau_{0,2}$ccon.) functions are independent.

iii. The relation between (FN-$\tau_{0,1}$ccon.) and (FN-$\tau_{0,2}$ccon.) functions are independent.

And we can show it by the following example.

Example 3.11:

i. 1- Take, **Example 3.9 (1)**. Then, f is (FN-ccon.) function.

But, f is not (FN-$\tau_{0,1}$ccon.) function. Since, from τ_x we get:

The family, $FN\tau_{x0,1} = \{0_N, 1_N, < x, (\frac{a}{0.9}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.1}, \frac{b}{0.4}) >\}$ is FNT.

Such that, 1_N-$FN\tau_{x0,1} = \{1_N, 0_N, < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) >\}$.

And, from τ_y we get:

The family, $FN\tau_{y0,1} = \{0_N, 1_N, < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,1}) \to (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.

If, $\eta_N = < y, (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) >$ is FN-open set in $FN\tau_{y0,1}$.

Then, $f^{-1}(\eta_N) = < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) > \notin 1_N$- $FN\tau_{x0,1}$.

2- Take, **Example 3.9 (2)**. Then, f is (FN-$\tau_{0,1}$ccon.) function.

But, f is not (FN-ccon.) function.

Since, 1_N-$\tau_x = \{1_N, 0_N, < x, (\frac{a}{0.5}, \frac{b}{0.7}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) >\}$.

Define, f : $(X, \tau_x) \to (Y, \tau_y)$ as follows: f(a) = b and f(b) = a.

If, $\beta_N = < y, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) >$ is FN-open set in τ_y.

Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.6}, \frac{b}{0.8}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}) > \notin 1_N$- τ_x.

ii. 1- Take, **Example 3.9 (1)**. Then, f is (FN-ccon.) function.

But, f is not (FN-$\tau_{0,2}$ccon.) function. Since, from τ_x we get:

The family, $FN\tau_{x0,2} = \{0_N, 1_N, < x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}) >\}$ is FNT.

Such that, 1_N-$FN\tau_{x0,2} = \{1_N, 0_N, < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.5}) >\}$.

And, from τ_y we get:

The family, $FN\tau_{y0,2} = \{0_N, 1_N, < y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,2}) \to (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.

If, $\Psi_N = < y, (\frac{a}{0.4}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.9}) >$ is FN-open set in $FN\tau_{y0,2}$.

Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.1}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.9}, \frac{b}{0.6}) > \notin 1_N$- $FN\tau_{x0,2}$.

2- Take, **Example 3.9 (3)**. Then, f is (FN-$\tau_{0,2}$ccon.) function.

But, f is not (FN-ccon.) function.

Since, 1_N-$\tau_x = \{1_N, 0_N, < x, (\frac{a}{0.8}, \frac{b}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}) >\}$.

Define f : $(X, \tau_x) \to (Y, \tau_y)$ as follows : f(a) = b and f(b) = a.

If, $\beta_N = < y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.2}) >$ is FN-open set in τ_y.

Then, $f^{-1}(\beta_N) = < x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}) > \notin 1_N$- τ_x.

iii. 1-Take, **Example 3.9 (2)**. Then, f is (FN-$\tau_{0,1}$ccon.) function.

But, f is not (FN-$\tau_{0,2}$ccon.) function.

Since, from τ_x we get:

The family, $FN\tau_{x0,2} = \{0_N, 1_N, < x, (\frac{a}{0.5}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) >\}$ is FNT.

Such that, 1_N-$FN\tau_{x0,2} = \{1_N, 0_N, < x, (\frac{a}{0.5}, \frac{b}{0.7}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.3}) >\}$.

And, from τ_y we get:

The family, $FN\tau_{y0,2} = \{0_N, 1_N, < y, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) >\}$ is FNT.

Define f: $(X, FN\tau_{x0,2}) \to (Y, FN\tau_{y0,2})$ as follows: f(a) = b and f(b) = a.

If, $\Psi_N = < y, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.3}) >$ is FN-open set in $FN\tau_{y0,2}$.

Then, $f^{-1}(\Psi_N) = < x, (\frac{a}{0.7}, \frac{b}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}) > \notin 1_N$-$FN\tau_{x0,2}$.

2- Take, **Example 3.9 (3)**. Then, f is (FN-$\tau_{0,2}$ccon.) function.

But, f is not (FN-$\tau_{0,1}$ccon.) function. Since, from τ_x we get:

The family, $FN\tau_{x0,1} = \{0_N, 1_N, < x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.7}) >\}$ is FNT.

Such that, 1_N-$FN\tau_{x0,1} = \{1_N, 0_N, < x, (\frac{a}{0.6}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.3}) >\}$.

And, from τ_y we get:

The family, $FN\tau_{y0,1} = \{0_N, 1_N, < y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.3}) >\}$ is FNT.

Define f : $(X, FN\tau_{x0,1}) \to (Y, FN\tau_{y0,1})$ as follows: f(a) = b and f(b) = a.

If, $\eta_N = < y, (\frac{a}{0.4}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.6}, \frac{b}{0.3}) >$ is FN-open set in $FN\tau_{y0,1}$.

Then, $f^{-1}(\eta_N) = < x, (\frac{a}{0.7}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}) > \notin 1_N$-$FN\tau_{x0,1}$.

Remark 3.12:

i. The relation between (FN-ccon.) and (FN-con.) are independent.
ii. The relation between (FN-$\tau_{0,1}$ccon.) and (FN-$\tau_{0,1}$con.) are independent.
iii. The relation between (FN-$\tau_{0,2}$ccon.) and (FN-$\tau_{0,2}$con.) are independent. And we can show it by the following example.

Example 3.13:

i. 1- Take, **Example 3.9 (1)**. Then, f is (FN-ccon.) function.
But, f is not (FN-con.) function. Since, $f^{-1}(\beta_N) \notin \tau_x$.
2- Take, **Example 3.2 (1)**. Then, f is (FN-con.) function.
But, f is not (FN-ccon.) function. Since, $f^{-1}(\beta_N) \notin 1_N\text{-}\tau_x$.
ii. 1-Take, **Example 3.9 (2)**. Then, f is (FN-$\tau_{0,1}$ccon.) function.
But, f is not (FN-$\tau_{0,1}$con.) function. Since, $f^{-1}(\eta_N) \notin FN\tau_{x0,1}$.
2- Take, **Example 3.2 (2)**. Then, f is (FN-$\tau_{0,1}$con.) function.
But, f is not (FN-$\tau_{0,1}$ccon.) function. Since, $f^{-1}(\eta_N) \notin 1_N\text{-} FN\tau_{x0,1}$.
iii. 1-Take, **Example 3.9 (3)**. Then, f is (FN-$\tau_{0,2}$ccon.) function.
But, f is not (FN-$\tau_{0,2}$con.) function. Since, $f^{-1}(\Psi_N) \notin FN\tau_{x0,2}$.
2- Take, **Example 3.2 (3)**. Then, f is (FN-$\tau_{0,2}$con.) function.
But, f is not (FN-$\tau_{0,2}$ccon.) function. Since, $f^{-1}(\Psi_N) \notin 1_N\text{-} FN\tau_{x0,2}$.

Definition 3.14:
Fuzzy neutrosophic subset λ_N of FNTS (X, τ) is called fuzzy neutrosophic-clopen set (FN-clopen, for short) set if λ_N is FN-closed set and FN-open set in same time.

Theorem 3.15: i. Let (X, τ_x) and (Y, τ_y) are two FNTSs and f: (X, τ_x) $\to$ (Y,τ_y) is a function. f is (FN-con.) iff f is (FN-ccon.) whenever, every the invers image of any FNS in τ_y is FN-clopen set in τ_x.
ii. Let (X, FN$\tau_{x0,1}$) and (Y, FN$\tau_{y0,1}$) are two FNTSs and f: (X,FN$\tau_{x0,1}$) $\to$ (Y, FN$\tau_{y0,1}$) is a function. f is (FN-$\tau_{0,1}$con.) iff f is (FN$\tau_{0,1}$ccon.) whenever, every the invers image of any FNS in FN$\tau_{y0,1}$ is FN-clopen set in FN$\tau_{x0,1}$.
iii. Let (X, FN$\tau_{x0,2}$) and (Y, FN$\tau_{y0,2}$) are two FNTSs and f : (X, FN$\tau_{x0,2}$) $\to$ (Y, FN$\tau_{y0,2}$) is a function.
f is (FN-$\tau_{0,2}$con.) iff f is (FN-$\tau_{0,2}$ccon.) whenever, every the invers image of any FNS in FN$\tau_{y0,2}$ is FN-clopen set in FN$\tau_{x0,2}$.

Proof: i. Let f be (FN-con.) function. If, β_N be FN-open set in τ_y.
Then, by **Definition 2.8** $f^{-1}(\beta_N) = \omega_N \in \tau_x$.
But, ω_N be FN-clopen set in τ_x. Therefore, $f^{-1}(\beta_N) = \omega_N \in 1_N\text{-}\tau_x$.
Hence, by **Definition 2.9** f is (FN-ccon.) function.
Conversely; the proof is direct.
ii. Let f be (FN-$\tau_{0,1}$con.) function. If, η_N be FN-open set in FN$\tau_{y0,1}$.
Then, by **Definition 3.1(i)** $f^{-1}(\eta_N) = \omega_N \in FN\tau_{x0,1}$.
But, ω_N be FN-clopen set in FN$\tau_{x0,1}$. So, $f^{-1}(\eta_N) = \omega_N \in 1_N\text{-} FN\tau_{x0,1}$.
Hence, by **Definition 3.8 (i)** f is (FN-$\tau_{0,1}$ccon.) function.
Conversely; the proof is direct.
iii. Let f be (FN-$\tau_{0,2}$con.) function. If, Ψ_N be FN-open set in FN$\tau_{y0,2}$.
Then, by **Definition 3.1(ii)** $f^{-1}(\Psi_N) = \omega_N \in FN\tau_{x0,2}$.
But, ω_N is FN-clopen set in FN$\tau_{x0,2}$. So, $f^{-1}(\Psi_N) = \omega_N \in 1_N\text{-} FN\tau_{x0,2}$.
Hence, by **Definition 3.8 (ii)** f is (FN-$\tau_{0,2}$ccon.) function.
Conversely; the proof is direct.

Remark 3.16: The next diagram showing the relationship between different functions. But the convers is not true in general.

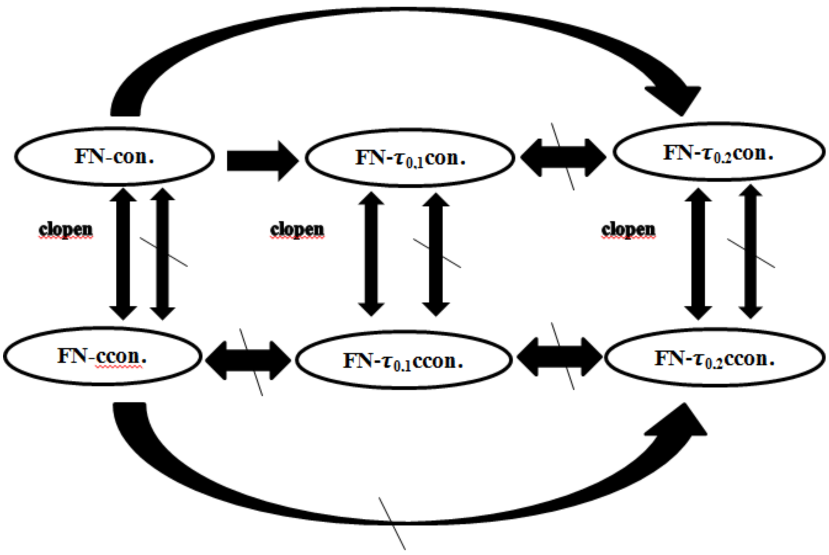

Diagram 3.1

References

[1] Zadeh, L. A. (1965). Fuzzy Sets. Inform. and Control, Vol.8: 338-353.
[2] Atanassov, K. and Stoeva, S. (1983). Intuitionistic Fuzzy Sets, in: Polish Syrup. On Interval & Fuzzy Mathematics, Poznan, proceedings: 23-26.
[3] Atanassov, K. (1986). Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, Vol. 20: 87-96.
[4] Atanassov, K. (1988). Review and new results on Intuitionistic Fuzzy Sets. Preprint IM-MFAIS, Sofia: 1-88.
[5] Smarandache, F. (1999). A Unifying Field in Logics Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press: Rehoboth, NM: 157 pp.
[6] Dhavaseelan, R. et al. (2018). Neutrosophic Generalized α-contra continuity, 27(2): 133-139.
[7] Salama, A. A. and Alblowi, S. A. (2012). Neutrosophic Set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics, 3(4): 31-35.
[8] Arockiarani, I. et al. (2013). Fuzzy Neutrosophic Soft topological spaces. IJMA, 4 (10): 225-238.
[9] Arockiarani, I. and Jency, J. M. (2014). More on Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Topological Spaces. IJIRS, 3(5): 642-652.
[10] Veereswari, Y. (2017). An Introduction to Fuzzy Neutrosophic Topological Spaces, IJMA. 8(3), 144-149.

بعض الانواع الجديدة من الدوال المستمرة من خلال فضاء تبولوجي نيوتروسوفك المضبب

فاطمة محمود محمد ، شيماء فائق مطر
قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث, عرفنا كل من لدوال المستمرة fuzzy neutrosophic-$\tau_{0,1}$, fuzzy neutrosophic-$\tau_{0,1}$contra, fuzzy neutrosophic-$\tau_{0,2}$, fuzzy neutrosophic-$\tau_{0,2}$contra. ثم وجدنا العلاقات بين الدوال المذكورة والمدروسة مع بعض المقارنات.